Prep. [3]

First Term

Algebra

Unit [1]

Lesson [1]

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Prep. [3] - First Term – Algebra – Unit [1] : Relations And Functions

Lesson [1] : Cartesian Product

The ordered pair

(a , b) is called an ordered pair

- a is called the first projection
- b is called the second projection

Remarks

- If a ≠ b , then (a , b) ≠ (b , a)

For example: (2 , 3) ≠ (3 , 2)

The equality of two ordered pairs

If (a , b) = (x , y) , then a = x , b = y

For example:

- If (a , b) = (3 , – 4) , then a = 3 , b = – 4
- If (x , 2) = (– 5 , y) , then x = – 5 , y = 2

The Cartesian product of two finite sets

For any two finite and non empty sets X and Y , we get :

The Cartesian product of the set X by the set Y and it is denoted by X × Y is the set of all ordered pairs whose first projection of each of them belongs to X and the second projection of each of them belongs to Y *i.e.* $X \times Y = \{(a , b) : a \in X , b \in Y\}$

For example:

1 If X = {1 , 2} , Y = {5 , 7 , 8} , then

X × Y = {1 , 2} × {5 , 7 , 8}

= {(1 , 5) , (1 , 7) , (1 , 8) , (2 , 5) , (2 , 7) , (2 , 8)}

	⊗	Second projection		
		5	**7**	**8**
First projection	**1**	(1 , 5)	(1 , 7)	(1 , 8)
	2	(2 , 5)	(2 , 7)	(2 , 8)

The opposite table helps us to get X × Y

Remark

The Cartesian product of the set X by itself and we denote it by $\boxed{X \times X}$ or by $\boxed{X^2}$ (it is read X two) is the set of all ordered pairs whose first projections and second projections belong both to X

i.e. $X \times X = \{(a , b) : a \in X , b \in X\}$

For example:

If $X = \{1 , 2\}$, then

$X \times X = \{1 , 2\} \times \{1 , 2\}$

$= \{(1 , 1) , (1 , 2) , (2 , 1) , (2 , 2)\}$

The opposite table helps us to get $X \times X$

	×	**Second projection** 1	**Second projection** 2
First projection	**1**	(1 , 1)	(1 , 2)
First projection	**2**	(2 , 1)	(2 , 2)

Remark

For any set X :

$X \times \varnothing = \varnothing \times X = \varnothing$ where $\varnothing$ is the empty set.

Representing the Cartesian product of two finite sets

We can represent the Cartesian product of two finite sets in two methods :

- **1st method :** The arrow diagram.
- **2nd method :** The graphical (Cartesian) diagram.

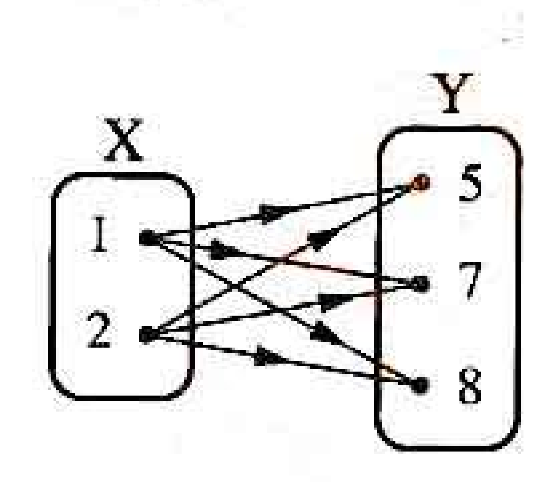
For example:

Let $X = \{1 , 2\}$, $Y = \{5 , 7 , 8\}$

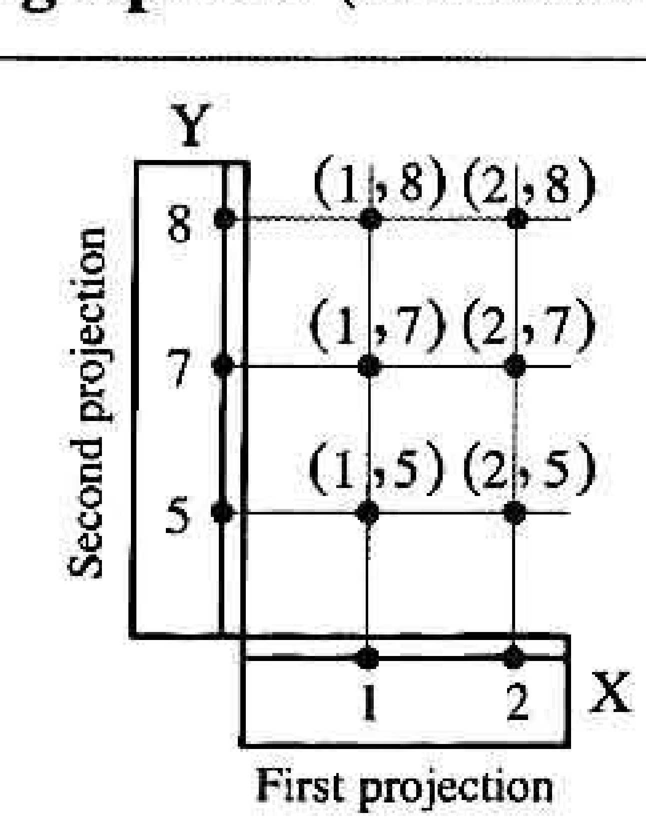
We can represent the Cartesian product X × Y where :

$\mathbf{X \times Y = \{(1 , 5) , (1 , 7) , (1 , 8) , (2 , 5) , (2 , 7) , (2 , 8)\}}$ **as follows :**

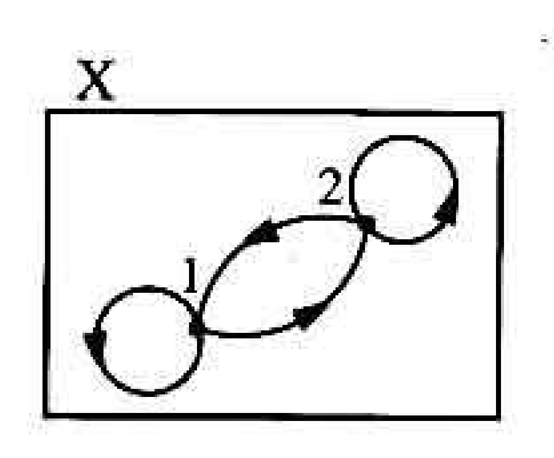
First : The arrow diagram



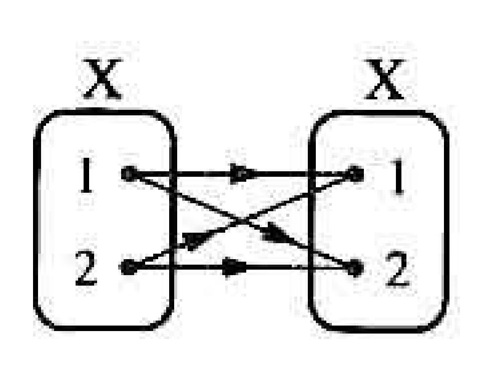
Second : The graphical (Cartesian) diagram



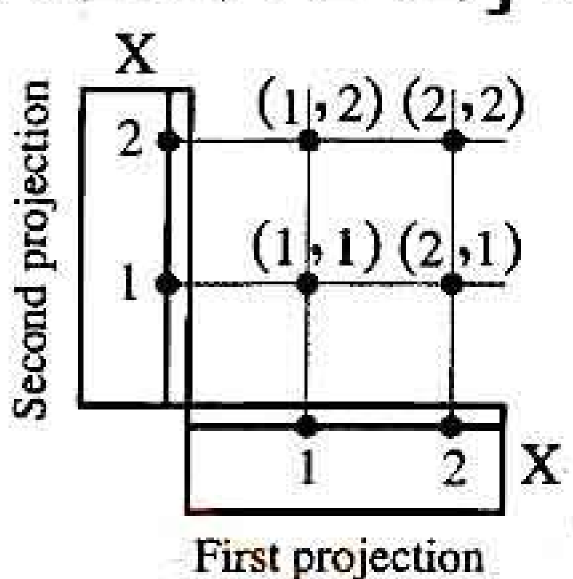
- Also , we can represent $X \times X$ where : $X \times X = \{(1 , 1) , (1 , 2) , (2 , 1) , (2 , 2)\}$ as follows :



or



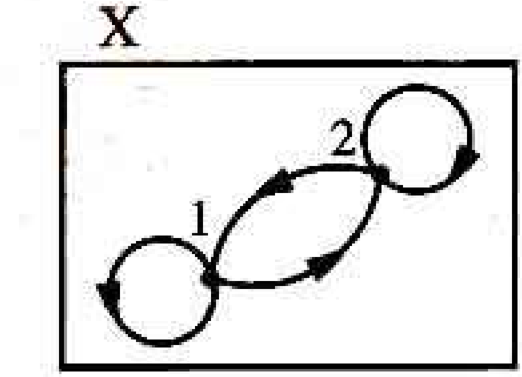
The arrow diagram



The Cartesian diagram

Remark

The ordered pairs in which the first projection equals the second projection in the previous Cartesian product (1 , 1) , (2 , 2) are represented in the arrow diagram by a loop ◯ to show that the arrow goes and returns to the same point.



The Cartesian product of two infinite sets

- We know that if X is a finite set (having n elements) , then the Cartesian product $X \times X$ is also a finite set (having n^2 elements).

For example: If n (X) = 3 , then n ($X \times X$) = 9

- But if X is an infinite set , then $X \times X$ is an infinite set also

As examples for that

$\mathbb{N} \times \mathbb{N} = \{(x , y) : x \in \mathbb{N} , y \in \mathbb{N}\}$, $\mathbb{Z} \times \mathbb{Z} = \{(x , y) : x \in \mathbb{Z} , y \in \mathbb{Z}\}$,

$\mathbb{Q} \times \mathbb{Q} = \{(x , y) : x \in \mathbb{Q} , y \in \mathbb{Q}\}$, $\mathbb{R} \times \mathbb{R} = \{(x , y) : x \in \mathbb{R} , y \in \mathbb{R}\}$

Representing the Cartesian product of two infinite sets

- We know that if X is a finite set , we represent the Cartesian product $X \times X$ graphically by a finite number of points.
- But if X is an infinite set , then the Cartesian product $X \times X$ represented graphically by an infinite number of points.

Remark [1]

First Quadrant	Second Quadrant	Third Quadrant	Fourth Quadrant	X – Axis	Y – Axis
(+ , +)	(– , +)	(– , –)	(+ , –)	(± , 0)	(0 , ±)

Remark [2]

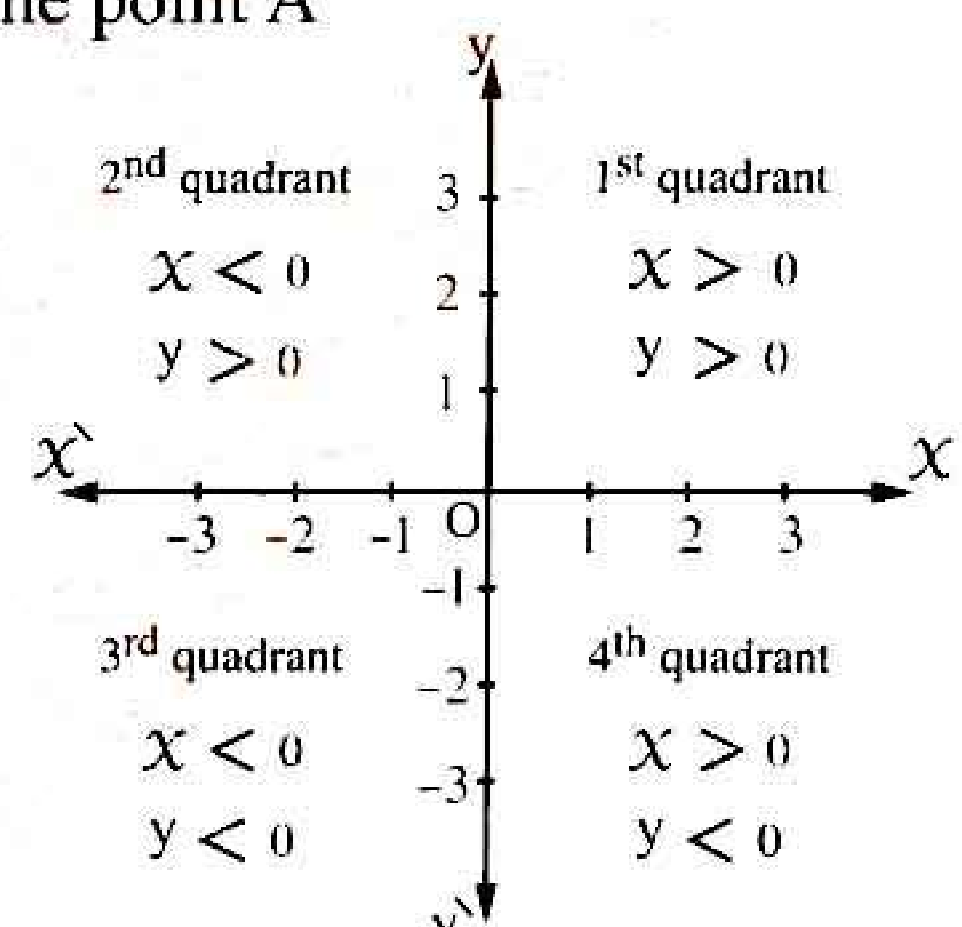
1 The horizontal straight line $\overleftrightarrow{XX}$ is called X-axis or the horizontal axis and the vertical straight line $\overleftrightarrow{yy}$ is called y-axis or the vertical axis.

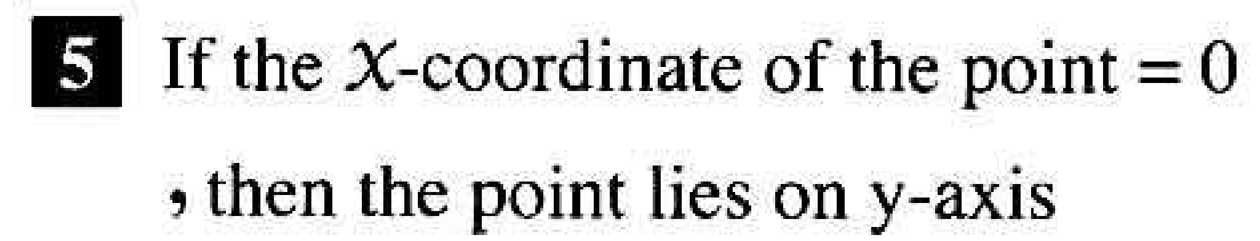
2 The point of intersection of the two axes $\overleftrightarrow{XX}$ and $\overleftrightarrow{yy}$ is called the origin point.

3 If the point A represents the ordered pair (X , y) in the Cartesian product $\mathbb{R} \times \mathbb{R}$, then :

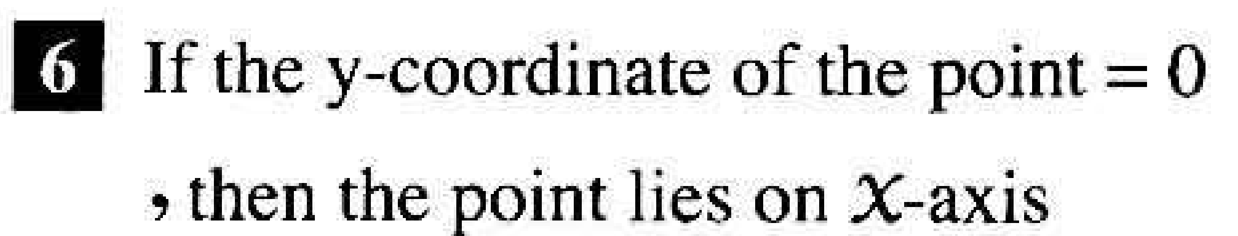
- The first projection X is called the X-coordinate of the point A
- The second projection y is called the y-coordinate of the point A

4 The two axes $\overleftrightarrow{XX}$ and $\overleftrightarrow{yy}$ divide the plane into four quadrants as shown in the opposite figure and we can determine the quadrant in which any point lies by knowing the signs of its two coordinates.




5 If the X-coordinate of the point = 0

, then the point lies on y-axis


6 If the y-coordinate of the point = 0

, then the point lies on X-axis

Exercises

[A] : Choose The Correct Answer : -

1	If A , B are two sets , then the set $\{(X , y) : X \in A , y \in B\}$ expresses …………… (a) n (A × B) (b) A × B (c) n (B × A) (d) B × A
2	If $(3 , 5) \in \{(3 , X) , (3 , 8) , (6 , 8)\}$, then X = …………… (a) 8 (b) 6 (c) 5 (d) 3
3	If X = {5} , Y = {3} , then n (X × Y) = …………… (a) 15 (b) 8 (c) 2 (d) 1
4	If X = {2} , Y = {1 , 3} , then n (X × Y) = …………… (a) 1 (b) 4 (c) 3 (d) 2
5	If X = {2} , Y = {0 , 4} , then n (X × Y) = …………… (a) 8 (b) 80 (c) 6 (d) 2
6	If n (X) = 2 , $n (Y^2) = 9$, then n (X × Y) = …………… (a) 6 (b) 18 (c) 11 (d) 7

7	If $n(X) = 3$, $Y = \{4 , 5\}$, then $n(X \times Y) =$ …………… (a) 2 (b) 6 (c) 5 (d) 3
8	If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ …………… (a) 4 (b) 9 (c) 15 (d) 36
9	If $n(X) = 5$, $n(X \times Y) = 10$, then $n(Y) =$ …………… (a) 4 (b) 3 (c) 2 (d) 1
10	If $n(X) = 5$, $n(X \times Y) = 15$, then $n(Y) =$ …………… (a) 3 (b) 5 (c) 15 (d) 8
11	If $n(X^2) = 4$, $n(X \times Y) = 6$, then $n(Y) =$ …………… (a) 2 (b) 3 (c) 4 (d) 6
12	If $X \times Y = \{(2 , 3) , (2 , 4)\}$, then $n(X) =$ …………… (a) 2 (b) 1 (c) 4 (d) 3
13	If $X = \{2 , 3 , 4\}$, then $n(X^2) =$ …………… (a) 3 (b) 6 (c) 9 (d) 12
14	If $X = \{7\}$, then $n(X^2) =$ …………… (a) 1 (b) 49 (c) 14 (d) 7
15	If $n(X) = 2$, $n(Y \times X) = 6$, then $n(Y^2) =$ …………… (a) 4 (b) 9 (c) 16 (d) 12
16	If $n(X) = 2$, $n(X \times Y) = 8$, then $n(Y^2) =$ …………… (a) 2 (b) 4 (c) 8 (d) 16
17	If $n(X \times Y) = 6$, $n(Y) = 2$, then $n(X^2) =$ …………… (a) 16 (b) 9 (c) 4 (d) 1
18	If $n(X^2) = 9$, $n(X \times Y) = 6$, then $n(Y^2) =$ …………… (a) 3 (b) 2 (c) 9 (d) 4
19	If $X \times Y = \{(1 , 2) , (1 , 3) , (1 , 4)\}$, then $n(X) + n(Y^2)$ …………… (a) 3 (b) 4 (c) 6 (d) 10
20	If $X = \{2\}$, $Y = \{3\}$, then $X \times Y =$ …………… (a) 6 (b) $\{6\}$ (c) $(2 , 3)$ (d) $\{(2 , 3)\}$

21	If $X = \{1\}$, then $X^2 =$ .... (a) 1 (b) (1 , 1) (c) $\{(1 , 1)\}$ (d) $\{1\}$
22	If $X = \{3\}$, then $X^2 =$ .... (a) $\{3 , 3\}$ (b) $\{(3 , 3)\}$ (c) $\{9\}$ (d) (3 , 3)
23	If $(2 , 5) \in \{3 , 2\} \times \{1 , x\}$, then $x =$ .... (a) 2 (b) 3 (c) 1 (d) 5
24	If $(3 , 5) \in \{3 , 6\} \times \{x , 8\}$, then $x =$ .... (a) 8 (b) 5 (c) 6 (d) 3
25	If $(1 , 4) \in \{1 , 5\} \times \{x , 7\}$, then $x =$ .... (a) 1 (b) 2 (c) 3 (d) 4
26	If $(x , 8) = (2 , x + y)$, then y = ... (a) 2 (b) 6 (c) 8 (d) 10
27	If $(2^x , 27) = (32 , y^3)$, then $\frac{x}{y} =$ .... (a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{32}{27}$ (d) $\frac{27}{32}$
28	If $(x - 1 , 3) = (1 , y + x)$, then y = ... (a) 1 (b) – 1 (c) 2 (d) – 2
29	If $(5 , x - 7) = (y + 1 , - 5)$, then $x + y =$ .... (a) 5 (b) – 1 (c) 6 (d) zero
30	If the point $(x , 7)$ lies on y-axis , then $5x + 1 =$ .... (a) zero (b) 1 (c) 5 (d) 6
31	If the point $(x - 3 , 2)$ lies on the y-axis , then $x =$ .... (a) 5 (b) 3 (c) – 3 (d) 0
32	If $(x + 1 , x - 3)$ lies on the x-axis , then $x =$ .... (a) – 1 (b) zero (c) – 2 (d) 3
33	If the point $(5 , b - 7)$ located on the x-axis, then b = ... (a) 2 (b) 5 (c) 7 (d) 12
34	If $(3 - x , x - 1)$ is located in the fourth quadrant where $x \in \mathbb{Z}$, then $x =$ .... (a) 4 (b) 3 (c) 2 (d) zero

35 If the point $(x - 3 , 2 - x)$ lies in the fourth quadrant , then $x =$ ……………

(a) 4 (b) 3 (c) 2 (d) 1

36 If the point $(x - 4 , 2 - x)$ lies on the fourth quadrant , where $x \in \mathbb{Z}$, then $x =$ ……………

(a) 2 (b) 3 (c) 4 (d) 5

37 If the point $(x - 5 , 7 - x)$ lies in the second quadrant , then $x =$ ……………

(a) 9 (b) 3 (c) 7 (d) 5

38 The point $(-2 , -3)$ lies on the …………… quadrant.

(a) first (b) second (c) third (d) fourth

39 The point $(-2 , 4)$ lies on the …………… quadrant.

(a) first (b) second (c) third (d) fourth

40 The point $(-3 , 4)$ lies in …………… quadrant.

(a) first (b) second (c) third (d) fourth

41 The point $(-3 , 4)$ lies in …………… quadrant.

(a) first (b) second (c) third (d) fourth

42 If $x \in \mathbb{R}_-$, then the point $\left(-x , \sqrt[3]{x}\right)$ lies in the …………… quadrant.

(a) first (b) second (c) third (d) fourth

43 If $b < 3$, then the point $(5 , b - 3)$ lies in the …………… quadrant.

(a) first (b) second (c) third (d) fourth

[C] : Essay Problems : -

1 If $X = \{2 , 5\}$, $Y = \{1 , 2\}$, $Z = \{3\}$

Find : (1) $n(X \times Z)$

(2) $(Y \cap X) \times Z$

Model Exam (2) Question (2) (a)

2 If $X = \{1 , 3\}$, $Y = \{3 , 5 , 7\}$ **Find :** $(X - Y) \times Y$ and $n(X \times Y)$

2018 Exam (15) Question (4) (a)

3 If $X = \{1\}$, $Y = \{2 , 3\}$, $Z = \{2 , 5 , 6\}$, **find :** $X \times (Y \cap Z)$

2018 Exam (22) Question (3) (b)

4	If $X = \{12\}$, $Y = \{4, 1\}$ and $Z = \{4, 5, -2\}$, **find :** (1) $X \times Y$ (2) $n(Y^2)$ (3) $(Y \cap Z) \times X$ 2017 Exam (1) Question (2) (a)
5	If $X = \{2, -1\}$, $Y = \{4, 0\}$, $Z = \{4, 5, -2\}$ **find :** (1) $Y \times X$ (2) $n(X \times Z)$ (3) $n(Z^2)$ 2018 Exam (14) Question (4) (b)
6	If $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$, **then find :** (1) $Z \times (X \cap Y)$ (2) $X - Y$ 2018 Exam (11) Question (2) (a)
7	If $X = \{1, 2, 3, 4\}$, $Y = \{3, 5, 6\}$, $Z = \{1, 2, 5, 6\}$ **Find :** (1) $(X \cap Y) \times Z$ (2) $(Z - X) \times Y$ 2018 Exam (17) Question (4) (b)
8	If $X \times Y = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$, **find :** (1) $X \cup Y$ (2) Y^2 2017 Exam (19) Question (4) (a)
9	If $X = \{1, 3, 5\}$ and R is a function on X where $R = \{(a, 3), (b, 1), (1, 5)\}$ **Find :** (1) X^2 (2) $n(X^2)$ (3) The value of $a + b$ 2017 Exam (9) Question (4) (a)
10	If $(x - 1, 11) = (8, y + 3)$, **find the value of :** $\sqrt{x + 2y}$ 2018 Exam (22) Question (2) (a)
11	If $(x - 1, 9) = (4, y^3 + 1)$, **find the values of :** x and y 2017 Exam (14) Question (3) (a)
12	If $(x + 3, 8) = (5, 2^y)$, then find the value of x and y 2018 Exam (2) Question (2) (a)
13	If $(x - 7, 28) = (-2, y^3 + 1)$, **find the value of :** $\sqrt[3]{x + y}$ 2017 Exam (19) Question (2) (a)
14	If $(2x, 4) = (8, y + 1)$ **find the value of :** $\sqrt{x^2 + y^2}$ 2018 Exam (14) Question (5) (a)

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Unit [1]

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Prep. [3] - First Term – Algebra – Unit [1] : Relations And Functions

Lesson [2] : Relation - function (mapping)

First The relation

Remarks

1 The relation R is a subset of the Cartesian product $X \times Y$ *i.e.* $R \subset X \times Y$

2 If $(a , b) \in$ the relation R , then we can express that by another method , we write "a R b" , it means that the element a is connected with the element b by the relation R

The conclusion

1 The relation from a set X to a set Y is a connection joining some or all the elements of X with some or all the elements of Y

2 If R is a relation from the set X to the set Y , then R is a set of ordered pairs where the first projection of each belongs to X and the second projection belongs to Y and the first projection connects with the second projection with respect to this relation.

3 The relation R from the set X to the set Y is a subset from the Cartesian product $X \times Y$ *i.e.* The relation $R \subset X \times Y$

Inversely : any subset of the Cartesian product $X \times Y$ expresses a relation from X to Y

4 The relation can be represented by an arrow diagram or by a Cartesian diagram (graphically).

Remark

If R is a relation from X to X , then : R is a relation on X and the relation $R \subset X \times X$

Second Functions (Mapping)

Generally

A relation from X to Y is said to be a function if :

1 In the relation , each element of the set X appears only once as a first projection in one of the ordered pairs of the relation. (Notice the relation R in the previous example)

2 In the arrow diagram which represents the relation , each element of X has one and only one arrow going out of it to one element of Y
(Notice the arrow diagram of the previous relation)

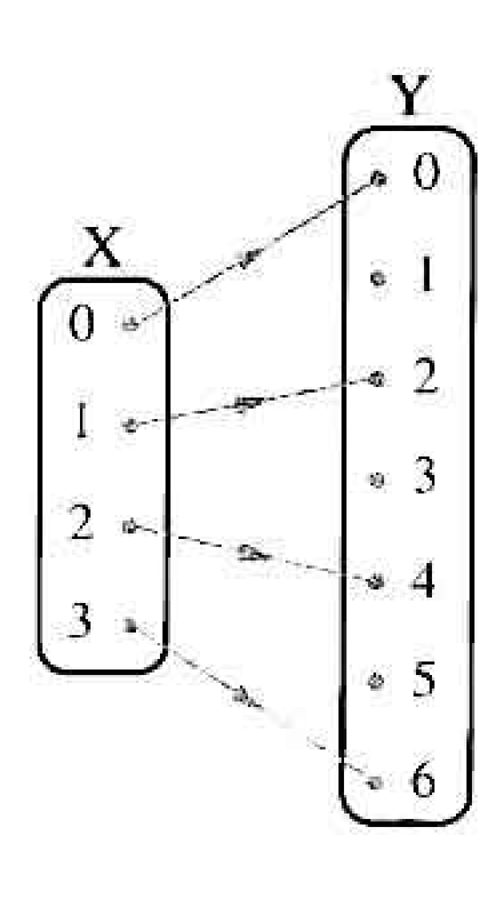
3 In the Cartesian diagram which represents the relation , each vertical line has one and only one point lying on it of the points which represent the relation.
(Notice the Cartesian diagram of the previous relation)

Introductory example

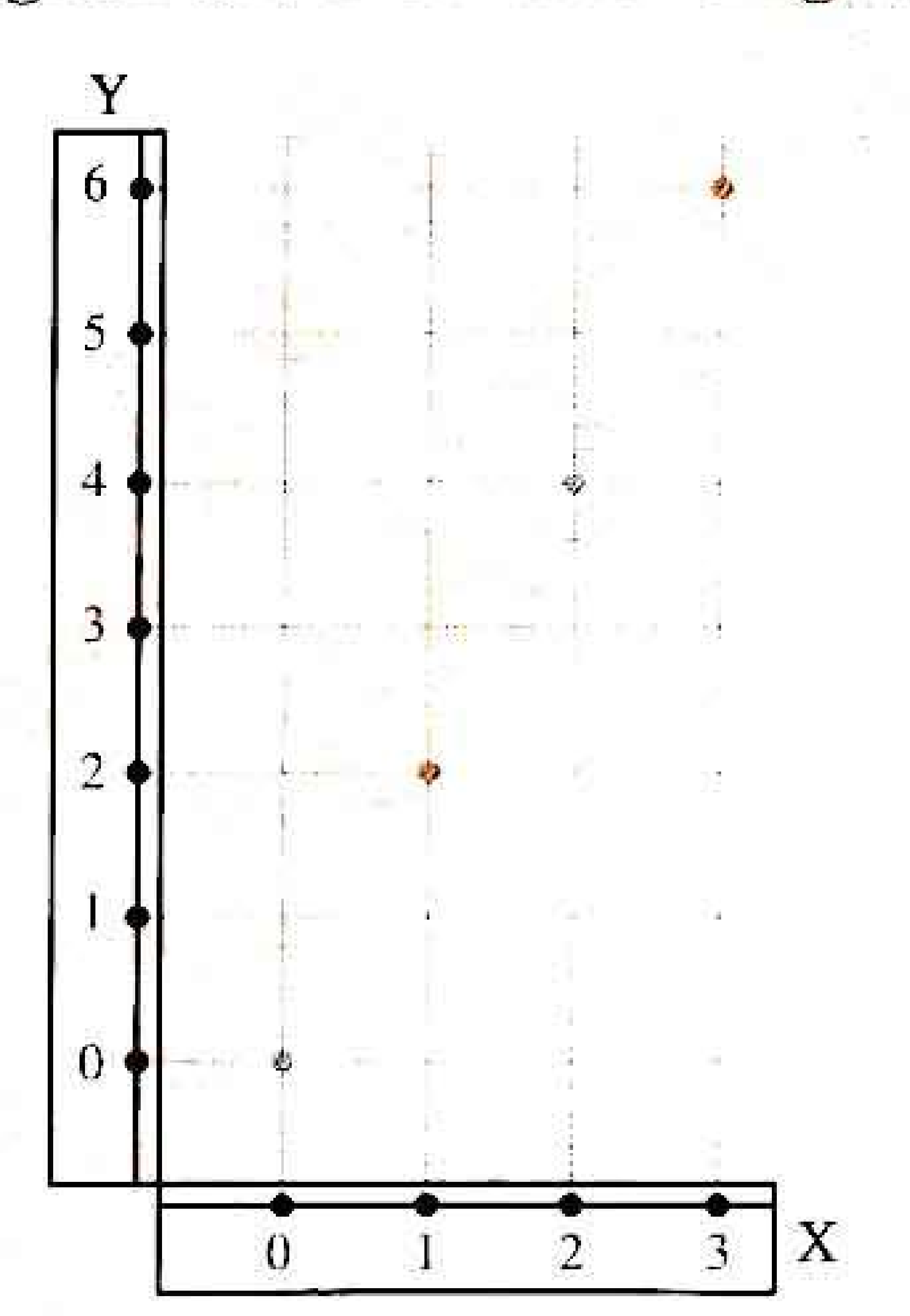
If X = {0 , 1 , 2 , 3} , Y = {0 , 1 , 2 , 3 , 4 , 5 , 6} and R is a relation from X to Y where "a R b" means "a = $\frac{1}{2}$ b " for each a ∈ X , b ∈ Y

Write R and represent it by an arrow diagram and a Cartesian diagram.

Solution R = {(0 , 0) , (1 , 2) , (2 , 4) , (3 , 6)}



The arrow diagram



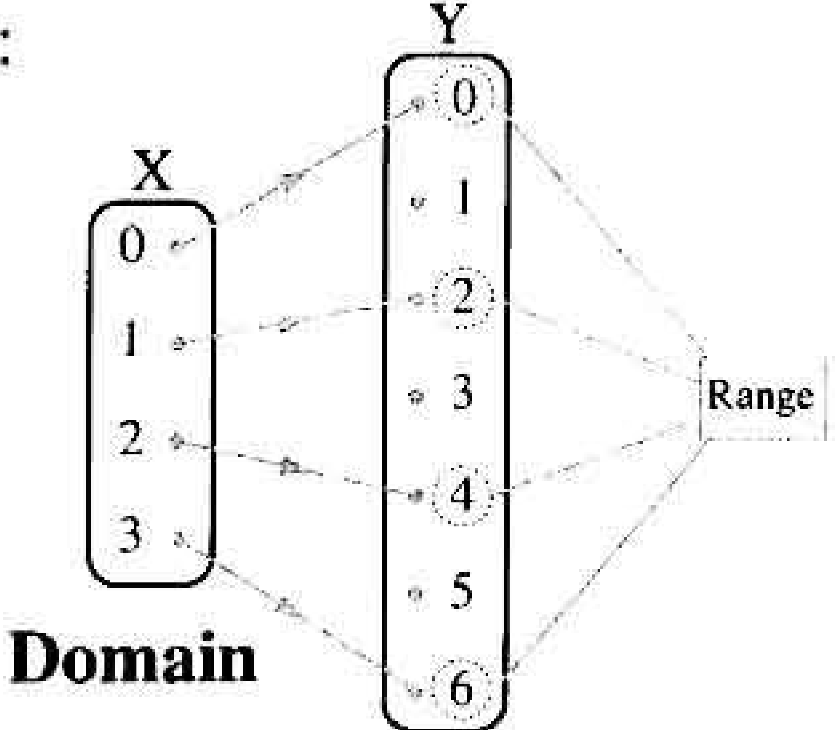
The Cartesian diagram

In the previous relation , we notice that :

Each element of the set X has been connected with **one and only one element** of the elements of the set Y

Such as , this relation is called a function or (mapping) , also :

- The set of X = {0 , 1 , 2 , 3} is called "the domain of the function".
- The set of Y = {0 , 1 , 2 , 3 , 4 , 5 , 6} is called "the codomain of the function".
- The set {0 , 2 , 4 , 6} is called "the range of the function" and it is a subset from the codomain of the function.



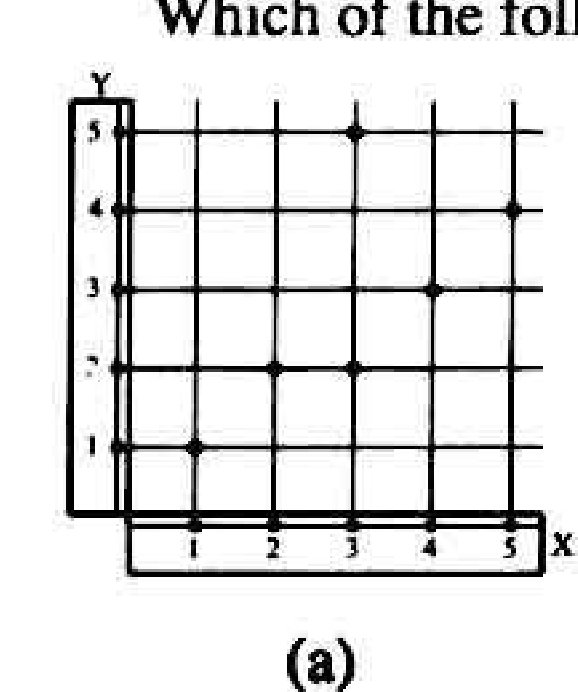
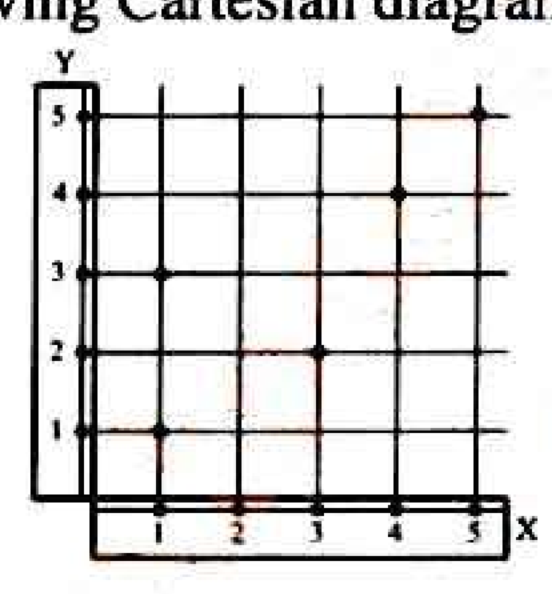
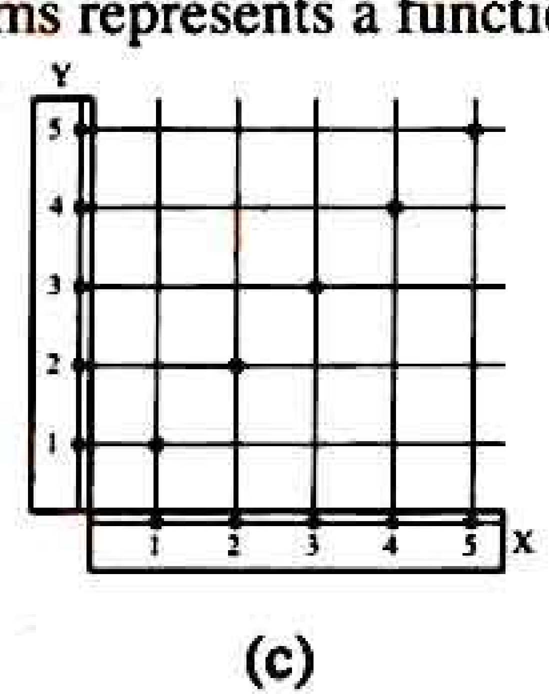
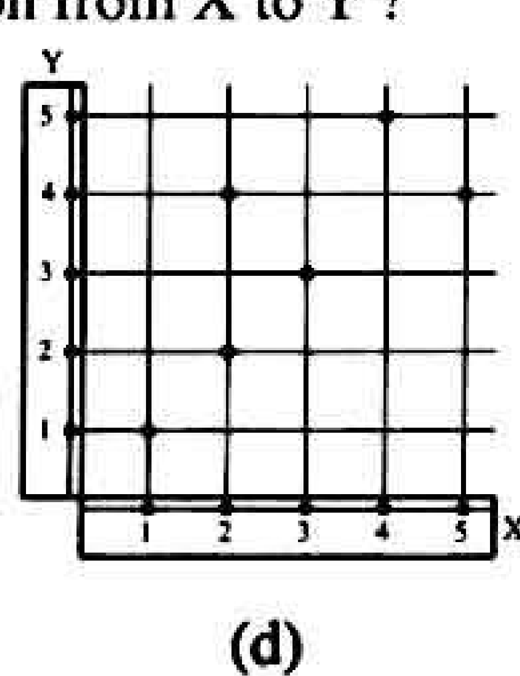
Prime numbers = { 2 , 3 , 5 , 7 , 11 , 13 , 17 , 19 , 23 , 29 , 31 , 37 }

Odd numbers = { 1 , 3 , 5 , 7 , 9 , 11 , 13 , 15 , 17 , 19 , ... }

Even numbers = { 0 , 2 , 4 , 6 , 8 , 10 , 12 , 14 , 16 , 18 , 20 , ... }

Exercises

[A] : Choose The Correct Answer : -

1	The set of images of the elements of the domain of the function is called .... (a) the rule. (b) the domain. (c) the range. (d) the codomain.
2	If the function $f : X \longrightarrow Y$, then the range of the function $f \subset$ .... (a) $X \times Y$ (b) X (c) $Y \times X$ (d) Y
3	If the relation $R = \{(4 , 3) , (1 , 3) , (2 , 5)\}$, then R represents a function when its range is .... (a) $\{4 , 1 , 2\}$ (b) $\{4 , 1 , 2 , 3 , 5\}$ (c) $\{3 , 5\}$ (d) $\mathbb{N}$
4	If $X = \{1 , 3 , 5\}$, $f : X \longrightarrow \mathbb{R}$ and $f(x) = 2x + 1$, then the set of images of the elements of the domain of the function f is .... (a) $\{3 , 5 , 11\}$ (b) $\{3 , 7 , 9\}$ (c) $\{1 , 3 , 11\}$ (d) $\{3 , 11 , 7\}$
5	Which of the following Cartesian diagrams represents a function from X to Y ?  (a)  (b)  (c)  (d)
6	The ordered pair that satisfies the relation $x + y = 3$ is .... (a) $(1 , -1)$ (b) $(1 , 2)$ (c) $(-1 , 1)$ (d) $(0 , 1)$
7	If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ .... (a) 4 (b) 9 (c) 15 (d) 36
8	If $X = \{2 , 3 , 4\}$, then $n(X^2) =$ .... (a) 3 (b) 6 (c) 9 (d) 12
9	If $n(X^2) = 9$, $n(X \times Y) = 6$, then $n(Y^2) =$ .... (a) 3 (b) 2 (c) 9 (d) 4
10	If $(2 , 5) \in \{3 , 2\} \times \{1 , x\}$, then $x =$ .... (a) 2 (b) 3 (c) 1 (d) 5

11	If the point (5 , b – 7) located on the $\mathcal{X}$-axis, then b = …………… (a) 2 (b) 5 (c) 7 (d) 12
12	The point (– 2 , – 3) lies on the …………… quadrant. (a) first (b) second (c) third (d) fourth
13	If ($\mathcal{X}$ – 1 , 3) = (1 , y + $\mathcal{X}$) , then y = …………… (a) 1 (b) – 1 (c) 2 (d) – 2
14	If X = {2 , 5} , which of the following arrow diagrams represents a function on the set X ? X X X X (a) (b) (c) (d)
15	If X = {1 , 2} , then the arrow diagram represents a function on X is …………… X X X X (a) (b) (c) (d)
16	If A , B are two sets , then the set {($\mathcal{X}$, y) : $\mathcal{X}$ ∈ A , y ∈ B} expresses …………… (a) n (A × B) (b) A × B (c) n (B × A) (d) B × A
17	If n (X) = 5 , n (X × Y) = 10 , then n (Y) = …………… (a) 4 (b) 3 (c) 2 (d) 1
18	If X = {7} , then n (X^2) = …………… (a) 1 (b) 49 (c) 14 (d) 7
19	If X × Y = {(1 , 2) , (1 , 3) , (1 , 4)} , then n (X) + n (Y^2) …………… (a) 3 (b) 4 (c) 6 (d) 10
20	If (3 , 5) ∈ {3 , 6} × {$\mathcal{X}$, 8} , then $\mathcal{X}$ = …………… (a) 8 (b) 5 (c) 6 (d) 3
21	If (5 , $\mathcal{X}$ – 7) = (y + 1 , – 5) , then $\mathcal{X}$ + y = …………… (a) 5 (b) – 1 (c) 6 (d) zero

22	If $(3 - x , x - 1)$ is located in the fourth quadrant where $x \in \mathbb{Z}$, then $x =$ …………… (a) 4 (b) 3 (c) 2 (d) zero
23	The point $(-2 , 4)$ lies on the …………… quadrant. (a) first (b) second (c) third (d) fourth
24	If $(3 , 5) \in \{(3 , x) , (3 , 8) , (6 , 8)\}$, then $x =$ …………… (a) 8 (b) 6 (c) 5 (d) 3
25	If $X = \{2\}$, $Y = \{0 , 4\}$, then $n(X \times Y) =$ …………… (a) 8 (b) 80 (c) 6 (d) 2
26	If $n(X) = 5$, $n(X \times Y) = 15$, then $n(Y) =$ …………… (a) 3 (b) 5 (c) 15 (d) 8
27	If $n(X) = 2$, $n(Y \times X) = 6$, then $n(Y^2) =$ …………… (a) 4 (b) 9 (c) 16 (d) 12
28	If $X = \{2\}$, $Y = \{3\}$, then $X \times Y =$ …………… (a) 6 (b) $\{6\}$ (c) $(2 , 3)$ (d) $\{(2 , 3)\}$
29	If $(1 , 4) \in \{1 , 5\} \times \{x , 7\}$, then $x =$ …………… (a) 1 (b) 2 (c) 3 (d) 4
30	If the point $(x , 7)$ lies on y-axis , then $5x + 1 =$ …………… (a) zero (b) 1 (c) 5 (d) 6
31	If the point $(x - 3 , 2 - x)$ lies in the fourth quadrant , then $x =$ …………… (a) 4 (b) 3 (c) 2 1
32	The point $(-3 , 4)$ lies in …………… quadrant. (a) first (b) second (c) third (d) fourth
33	If $X = \{5\}$, $Y = \{3\}$, then $n(X \times Y) =$ …………… (a) 15 (b) 8 (c) 2 (d) 1
34	If $n(X) = 2$, $n(Y^2) = 9$, then $n(X \times Y) =$ …………… (a) 6 (b) 18 (c) 11 (d) 7
35	If $n(X^2) = 4$, $n(X \times Y) = 6$, then $n(Y) =$ …………… (a) 2 (b) 3 (c) 4 (d) 6

36	If $n(X) = 2$, $n(X \times Y) = 8$, then $n(Y^2) =$ .... (a) 2 (b) 4 (c) 8 (d) 16
37	If $X = \{1\}$, then $X^2 =$ .... (a) 1 (b) $(1, 1)$ (c) $\{(1, 1)\}$ (d) $\{1\}$
38	If $(x, 8) = (2, x + y)$, then $y =$ .... (a) 2 (b) 6 (c) 8 (d) 10
39	If the point $(x - 3, 2)$ lies on the y-axis , then $x =$ .... (a) 5 (b) 3 (c) -3 (d) 0
40	If the point $(x - 4, 2 - x)$ lies on the fourth quadrant, where $x \in \mathbb{Z}$, then $x =$ .... (a) 2 (b) 3 (c) 4 (d) 5
41	The point $(-3, 4)$ lies in .... quadrant. (a) first (b) second (c) third (d) fourth
42	If $X = \{2\}$, $Y = \{1, 3\}$, then $n(X \times Y) =$ .... (a) 1 (b) 4 (c) 3 (d) 2
43	If $n(X) = 3$, $Y = \{4, 5\}$, then $n(X \times Y) =$ .... (a) 2 (b) 6 (c) 5 (d) 3
44	If $X \times Y = \{(2, 3), (2, 4)\}$, then $n(X) =$ .... (a) 2 (b) 1 (c) 4 (d) 3
45	If $n(X \times Y) = 6$, $n(Y) = 2$, then $n(X^2) =$ .... (a) 16 (b) 9 (c) 4 (d) 1
46	If $X = \{3\}$, then $X^2 =$ .... (a) $\{3, 3\}$ (b) $\{(3, 3)\}$ (c) $\{9\}$ (d) $(3, 3)$
47	If $(2^x, 27) = (32, y^3)$, then $\frac{x}{y} =$ .... (a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{32}{27}$ (d) $\frac{27}{32}$
48	If $(x + 1, x - 3)$ lies on the x-axis , then $x =$ .... (a) -1 (b) zero (c) -2 (d) 3
49	If the point $(x - 5, 7 - x)$ lies in the second quadrant, then $x =$ .... (a) 9 (b) 3 (c) 7 (d) 5

[C] : Essay Problems : -

1	If $X = \{0, 1, 2, 3\}$, $Y = \{-1, 0, 1, 4, 9\}$ and R is a relation from X to Y where "a R b" means "$a = \sqrt{b}$" for all $a \in X$, $b \in Y$, write R and represent it by an arrow diagram. Is R a function ? Why ? 2017 Exam (14) Question (2) (b)
2	If $X = \{0, 1, 3\}$, $Y = \{2, 3, 4, 5, 6, 7\}$ and R is a relation from X to Y where "a R b" means" $b = 5 - a$" for all $a \in X$, $b \in Y$ (1) Write the relation R (2) Mention giving reasons if R is a function from X to Y or not , and if it is a function , find its range. 2017 Exam (16) Question (2) (a)
3	If $X = \{1, 2, 3\}$, $Y = \{-1\}$ and R is a relation from X to Y where "a R b" means "$a + b \geq 1$" for all $a \in X$, $b \in Y$ Write the relation R and represent it by an arrow diagram. Is R a function ? and why ? 2018 Exam (1) Question (4) (b)
4	If $X = \{1, 2, 3\}$, $Y = \{1, 2, 4, 9\}$ and R is a relation from X to Y where "a R b" means "$a^2 = b$" for all $a \in X$, $b \in Y$ (1) Write R and represent it by an arrow diagram. (2) Is R a function ? Why ? (3) If R is a function find its range. 2018 Exam (12) Question (2) (a)
5	If $X = \{1, 2, 4, 5\}$, $Y = \{1, 4, 16\}$ and R is a relation from X to Y where "a R b" means "$a^2 = b$" for all $a \in X$ and $b \in Y$ (1) Find the relation R (2) Represent the relation R by an arrow diagram. (3) Is R a function ? Why ? 2018 Exam (10) Question (2) (a)
6	If $X = \{1, 4, 7\}$, $Y = \{-1, 1, 4, 7\}$ and R is a relation from X to Y , where "a R b" means "$a + \|b\| = 8$" for each $a \in X$, $b \in Y$ (1) Write the relation and represent it by an arrow diagram. (2) Mention giving reasons if R is a function from X to Y or not.

	2017 Exam (8) Question (3) (b)
7	If $X = \{1, 2, 5\}$, $Y = \{2, 3, 7, 8\}$ and R is a relation from X to Y where "a R b" means "a + b = an odd number" for all $a \in X$, $b \in Y$ (1) Write R and represent it by an arrow diagram. (2) Show that R is a function. Why ? 2018 Exam (17) Question (2) (a)
8	If $X = \{2, 3, 4\}$, $Y = \{y : y \in \mathbb{N}, 2 < y \leq 9\}$ where $\mathbb{N}$ is the set of natural numbers and R is a relation from X to Y where a R b means "2 a = b" for each of $a \in X$, $b \in Y$, write R and represent it by and arrow diagram. Show that R is a function from X to Y and find its range. 2018 Exam (18) Question (2) (a)
9	If $X = \{-1, 1, 0, 2\}$, R is a relation on X , where "a R b" means "$b = a^2$" for each $(a, b) \in X^2$ (1) Write the relation R and represent it by an arrow diagram. (2) Is R a function ? And why ? 2017 Exam (5) Question (4) (a)
10	If $X = \{2, 3, 5\}$, $Y = \{4, 6, 8, 10\}$ and R is a relation form X to Y where a R b means "2 a = b" for all $a \in X$, $b \in Y$ (1) Write R and represent it by an arrow diagram. (2) Show that R is a function. Model Exam (1) Question (3) (a)
11	If $X = \{-2, -1, 0, 1, 2\}$ and R is a given relation on X where "a R b" means "The number a is the additive inverse of the number b" for each of $a \in X$, $b \in X$ Write the relation R and represent it by an arrow diagram. Is R a function ? and why ? 2018 Exam (13) Question (2) (a)
12	If $X = \{4, 6, 8, 10\}$, $Y = \{2, 3, 4, 5\}$ and R is a relation from X to Y, where "a R b" means "a = 2 b" for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram. Is R a function ? Why ? 2017 Exam (4) Question (2) (a)

Prep. [3]

First Term

Algebra

Unit [1]

Lesson [3]

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Prep. [3] - First Term – Algebra – Unit [1] : Relations And Functions

Lesson [3] : Polynomial Functions - Part (1)

Remark [1]

The mathematical form $f(X) = X^2$ is called the rule of the function f, and it is used to find the image of any element of the domain by the function f

Remark [2]

- **If f is a function from the set X to the set Y i.e. $f : X \longrightarrow Y$, then :**

1 X is called the **domain** of the function f

2 Y is called the **codomain** of the function f

3 The set of images of the elements of the set X by the function f is called the **range** of the function f which is a subset of the codomain Y

Remark [3]

If f is a function from the set X to itself :
i.e. $f : X \longrightarrow X$, then we say «f is a function on X»

Polynomial functions

Definition

The function $f : R \longrightarrow R$, $f(X) = a_0 + a_1 X + a_2 X^2 + \cdots\cdots + a_n X^n$
where $a_0, a_1, a_2, \cdots\cdots, a_n \in \mathbb{R}$, $n \in \mathbb{N}$ is called a polynomial function.

i.e. The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified :

1 Each of the domain and the codomain of the function is the set of real numbers.

2 The power (the index) of the variable X in any of its terms is a natural number.

For example: The following functions are all polynomial functions :

- $f : f(X) = 2X + 5$
- $g : g(X) = X^2 - 2X + 1$
- $k : k(X) = 8$
- $n : n(X) = 1 + \sqrt{2}X - 9X^3$

Remark [4]

If the domain or the codomain of a function is not the set of real numbers , then that function is not a polynomial function.

Remark [5]

When we search if the function is a polynomial or not , we do not simplify its rule.

For example:

The function $f_1 : f_1(x) = x\left(x + \frac{1}{x}\right)$ doesn't represent a polynomial function because $f_1(0) \notin \mathbb{R}$ while the function $f_2 : f_2(x) = x^2 + 1$ represents a polynomial function.

And notice that: $x\left(x + \frac{1}{x}\right) = x^2 + 1$ for all real numbers except 0

The degree of the polynomial function

The degree of the polynomial function is the highest power of the variable in the function rule.

For example:

- The function $f_1 : f_1(x) = 3x - \frac{1}{2}$ is of the first degree (a linear function)
- The function $f_2 : f_2(x) = \sqrt{5}\,x^2 - 3x + 4$ is of the second degree (quadratic function)
- The function $f_3 : f_3(x) = x^3 - 5x^2 + 4$ is of the third degree (cubic function)

Remark [6]

The function $f : f(x) = a$ where $a \in \mathbb{R} - \{0\}$
is a polynomial function of zero degree. (a constant function) as $f(x) = 3$
In the case of $a = 0$ ***i.e.*** when $f(x) = 0$, then the function has no degree.

Exercises

[A] : Choose The Correct Answer : -

1. If $f(X) = 3$, then $3f(2) - 2f(3) =$ ....
   (a) zero   (b) 4   (c) 1   3

2. If f is a function such that $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 3$, then $\frac{f(6)}{f(\text{zero})} =$ ....
   (a) 6   (b) 1   (c) 3   (d) undefined.

3. If $f(X) = 5$, then $f(3) - f(-3) =$ ....
   (a) zero   (b) 10   (c) 6   (d) -6

4. If $f(X) = 5$ is represented by a straight line parallel to the X-axis, then this line passes through the point ....
   (a) $(0 , 5)$ (b) $(5 , 0)$ (c) $(5 , -5)$ (d) $(0 , 0)$

5. If $f(X) = 7$, then $f(-3) =$ ....
   (a) 7   (b) -7 (c) 21   (d) -21

6. If $f(X) = 2X + b$ and $f(5) = 11$, then b = ...
   (a) 3   (b) 2   (c) 1   (d) zero

7. The function f where $f(X) = -2X$ is represented graphically by a straight line passing through the point ....
   (a) $(-2 , 0)$ (b) $(0 , -2)$ (c) $(0 , 0)$ (d) $(-2 , -2)$

8. If the straight line which represents the function $f : f(X) = 2X - a$ passes through the origin point , then a = ...
   (a) -2 (b) 2   (c) 0   (d) 3

9. If the point $(a , 5)$ lies on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 3X - 4$, then a = ...
   (a) 3   (b) -3 (c) 1   (d) -1

10. The function $f : f(X) = 3X$ is represented graphically by a straight line passing through the point ....
    (a) $(3 , 3)$ (b) $(3 , 0)$ (c) $(0 , 0)$ (d) $(0 , 3)$

11. If the point $(2 , a - 1)$ lies on the straight line represented to the function $f : f(X) = 4X - 5$, then a = ...
    (a) 4   (b) 1   (c) 3   (d) 2

12 If $f(X) = 4X + b$, $f(3) = 15$, then b = ...

(a) 6 (b) 3 (c) 4 (d) – 3

13 If $(2, -6) \in f$ where $f(X) = kX + 8$, then k = ...

(a) 16 (b) 7 (c) – 7 (d) 2

14 The function f where $f(X) = X^2 - (X^2 - 3X)$ is polynomial of the .... degree.

(a) first (b) second (c) third (d) fourth

15 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = X^3 - (X^3 - 3)$, then the degree of the function is the ....

(a) first (b) second (c) third (d) zero

16 If $f(X) = X^3$, then $f(1) + f(-1) =$ ....

(a) 2 (b) – 2 (c) zero (d) 4

17 If $f(X) = X^3$, then $f(2) + f(-2) =$ ....

(a) zero (b) 16 (c) – 16 (d) 4

18 If $f(X) = X^2 - 2X$, then $f(3) =$ ....

(a) – 3 (b) 3 (c) 0 (d) 15

19 If $f(X) = nX^2 + 2X^n - 3$, then the possible set of values of n such that f is a function of the second degree is ....

(a) $\{2, 3\}$ (b) $\{1, -1\}$ (c) $\{2, 1, 0\}$ (d) $\{2, 1\}$

20 The function $f : \mathbb{R} \longrightarrow \mathbb{R} : f(X) = a^2 X + a$, where $a \neq 0$ is a polynomial of the .... degree.

(a) first (b) second (c) third (d) fourth

21 The set of images of the elements of the domain of the function is called ....

(a) the rule. (b) the domain. (c) the range. (d) the codomain.

22 If $X = \{1, 2\}$, then the arrow diagram represents a function on X is ....

(a) (b) (c) (d)

23 If $n(X) = 3$, $Y = \{4, 5\}$, then $n(X \times Y) =$ ....

(a) 2 (b) 6 (c) 5 (d) 3

24	If $X = \{7\}$, then $n(X^2) =$ .... (a) 1 (b) 49 (c) 14 (d) 7
25	If $X = \{1\}$, then $X^2 =$ .... (a) 1 (b) $(1, 1)$ (c) $\{(1, 1)\}$ (d) $\{1\}$
26	The point $(-3, 4)$ lies in .... quadrant. (a) first (b) second (c) third (d) fourth
27	If $X = \{2, 5\}$, which of the following arrow diagrams represents a function on the set X ? X X X X (a) (b) (c) (d)
28	If $n(X) = 2$, $n(Y^2) = 9$, then $n(X \times Y) =$ .... (a) 6 (b) 18 (c) 11 (d) 7
29	If $X = \{2, 3, 4\}$, then $n(X^2) =$ .... (a) 3 (b) 6 (c) 9 (d) 12
30	If $X = \{2\}$, $Y = \{3\}$, then $X \times Y =$ .... (a) 6 (b) $\{6\}$ (c) $(2, 3)$ (d) $\{(2, 3)\}$
31	The point $(-2, 4)$ lies on the .... quadrant. (a) first (b) second (c) third (d) fourth
32	The ordered pair that satisfies the relation $x + y = 3$ is .... (a) $(1, -1)$ (b) $(1, 2)$ (c) $(-1, 1)$ (d) $(0, 1)$
33	If $X = \{2\}$, $Y = \{0, 4\}$, then $n(X \times Y) =$ .... (a) 8 (b) 80 (c) 6 (d) 2
34	If $X \times Y = \{(2, 3), (2, 4)\}$, then $n(X) =$ .... (a) 2 (b) 1 (c) 4 (d) 3
35	If $X \times Y = \{(1, 2), (1, 3), (1, 4)\}$, then $n(X) + n(Y^2)$ .... (a) 3 (b) 4 (c) 6 (d) 10
36	The point $(-2, -3)$ lies on the .... quadrant. (a) first (b) second (c) third (d) fourth

37	
38	Which of the following Cartesian diagrams represents a function from X to Y ? (a) (b) (c) (d)
39	If X = {2} , Y = {1 , 3} , then n (X × Y) = …………… (a) 1 (b) 4 (c) 3 (d) 2
40	If $n(X^2) = 4$, n (X × Y) = 6 , then n (Y) = …………… (a) 2 (b) 3 (c) 4 (d) 6
41	If $n(X^2) = 9$, n (X × Y) = 6 , then $n(Y^2)$ = …………… (a) 3 (b) 2 (c) 9 (d) 4
42	If $(3 - x , x - 1)$ is located in the fourth quadrant where $x \in \mathbb{Z}$, then x = …………… (a) 4 (b) 3 (c) 2 (d) zero
43	
44	If X = {1 , 3 , 5} , f : X ⟶ ℝ and $f(x) = 2x + 1$, then the set of images of the elements of the domain of the function f is …………… (a) {3 , 5 , 11} (b) {3 , 7 , 9} (c) {1 , 3 , 11} (d) {3 , 11 , 7}
45	If X = {5} , Y = {3} , then n (X × Y) = …………… (a) 15 (b) 8 (c) 2 (d) 1
46	If n (X) = 5 , n (X × Y) = 15 , then n (Y) = …………… (a) 3 (b) 5 (c) 15 (d) 8
47	If n (X × Y) = 6 , n (Y) = 2 , then $n(X^2)$ = …………… (a) 16 (b) 9 (c) 4 (d) 1
48	If the point (5 , b – 7) located on the x-axis, then b = …………… (a) 2 (b) 5 (c) 7 (d) 12
49	If b < 3 , then the point (5 , b – 3) lies in the …………… quadrant. (a) first (b) second (c) third (d) fourth

50	If the relation R = $\{(4, 3), (1, 3), (2, 5)\}$, then R represents a function when its range is .... (a) $\{4, 1, 2\}$ (b) $\{4, 1, 2, 3, 5\}$ (c) $\{3, 5\}$ (d) $\mathbb{N}$
51	If $(3, 5) \in \{(3, x), (3, 8), (6, 8)\}$, then x = ... (a) 8 (b) 6 (c) 5 (d) 3
52	If n (X) = 5 , n (X × Y) = 10 , then n (Y) = ... (a) 4 (b) 3 (c) 2 (d) 1
53	If n (X) = 2 , n (X × Y) = 8 , then n (Y^2) = ... (a) 2 (b) 4 (c) 8 (d) 16
54	If $(x + 1, x - 3)$ lies on the x-axis , then x = ... (a) – 1 (b) zero (c) – 2 (d) 3
55	If $x \in \mathbb{R}_-$, then the point $(-x, \sqrt[3]{x})$ lies in the .... quadrant. (a) first (b) second (c) third (d) fourth
56	If the function f : X ⟶ Y , then the range of the function $f \subset$ .... (a) X × Y (b) X (c) Y × X (d) Y
57	If A , B are two sets , then the set $\{(x, y) : x \in A, y \in B\}$ expresses .... (a) n (A × B) (b) A × B (c) n (B × A) (d) B × A
58	If n (X) = 3 , n (X × Y) = 12 , then n (Y) = ... (a) 4 (b) 9 (c) 15 (d) 36
59	If n (X) = 2 , n (Y × X) = 6 , then n (Y^2) = ... (a) 4 (b) 9 (c) 16 (d) 12
60	If X = $\{3\}$, then X^2 = ... (a) $\{3, 3\}$ (b) $\{(3, 3)\}$ (c) $\{9\}$ (d) (3 , 3)
61	The point (– 3 , 4) lies in .... quadrant. (a) first (b) second (c) third (d) fourth

[C] : Essay Problems : -

1	If $f(X) = X^2 - X + 3$, **then find :** $f(-2) + f(2)$ 2017 Exam (3) Question (4) (b)
2	If f and R are two functions , where $f(X) = -2X + 3$ and $R(X) = -7$, find the degree of f , and calculate the value of : $f(0) \times R(0)$ 2017 Exam (5) Question (2) (a)
3	If $f(X) = 4X + b$, $f(3) = 15$, **find :** (1) The value of b (2) The value of $f(2) + f(5)$ 2017 Exam (20) Question (4) (b)
4	If the point (a , 8) lies on the straight line which represents the function f : $f(X) = 3X - 7$, then find the value of a 2018 Exam (2) Question (4) (a)
5	If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X - a$ cuts y-axis at the point (b , 3) , **then find the values of :** a and b 2017 Exam (14) Question (5) (a)
6	If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(X) = 6X - a$ cuts y-axis at the point (b , 3) , then find the value of a and b 2018 Exam (19) Question (4) (a)
7	If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R} : f(X) = aX + b$ intersects the X-axis at the point (3 , 0) and intersects the y-axis at the point (0 , – 3) , then find the value of the two constants a and b and find the value of $f(1)$ 2018 Exam (5) Question (3) (b)
8	If the straight line represented to the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 6X - 9k$ cuts X-axis at the point (6 , m – 2). Find the value of m , k 2018 Exam (7) Question (4) (a)
9	If $X = \{1, 3, 5\}$, $f : X \longrightarrow \mathbb{R}$ and $f(X) = 3X$ **, find :** (1) the range of f (2) $f(6)$ 2017 Exam (10) Question (3) (a)

Prep. [3]

First Term

Algebra

Unit [1]

Lesson [4]

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Prep. [3] - First Term – Algebra – Unit [1] : Relations And Functions

Lesson [4] : Polynomial Functions - Part (2)

First The linear function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ where $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is called a linear function (it is a polynomial function of the first degree).

Examples of linear functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = X - 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 2X + 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 3X$

Notice that :

- In each of the shown functions , the index of X is 1 , therefore each of them is a function of the first degree.

The graphical representation of the linear function

- The linear function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is represented graphically by a straight line intersecting :

  – The y-axis at the point (0 , b) – The X-axis at the point $\left(\frac{-b}{a}, 0\right)$

- To represent a linear function , it is enough to find two ordered pairs belonging to the function.
- You can find a third ordered pair to check that the three points are on the same straight line.

Generally

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX$, $a \in \mathbb{R}^*$

is represented graphically by a straight line passing through the origin point (0 , 0)

Second The constant function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = b$, $b \in \mathbb{R}$ is called a constant function.

For example:

$f : f(X) = 5$ is a constant function where

$f(1) = 5$, $f(0) = 5$, $f(-2) = 5$, ··· and so on.

Graphical representation of the constant function

The constant function $f : f(X) = b$ (where $b \in \mathbb{R}$) is represented by a straight line parallel to X-axis and passes through the point (0 , b) this line is :

- above X-axis if $b > 0$
- below X-axis if $b < 0$
- coincident with X-axis if $b = 0$

Third The quadratic function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX^2 + bX + c$
where a , b and c are real numbers , $a \neq 0$
is called a quadratic function (it is a polynomial function of the second degree).

Examples of quadratic functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = X^2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = X^2 - 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 3X^2 - 7X + 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 6 - X^2 + X$

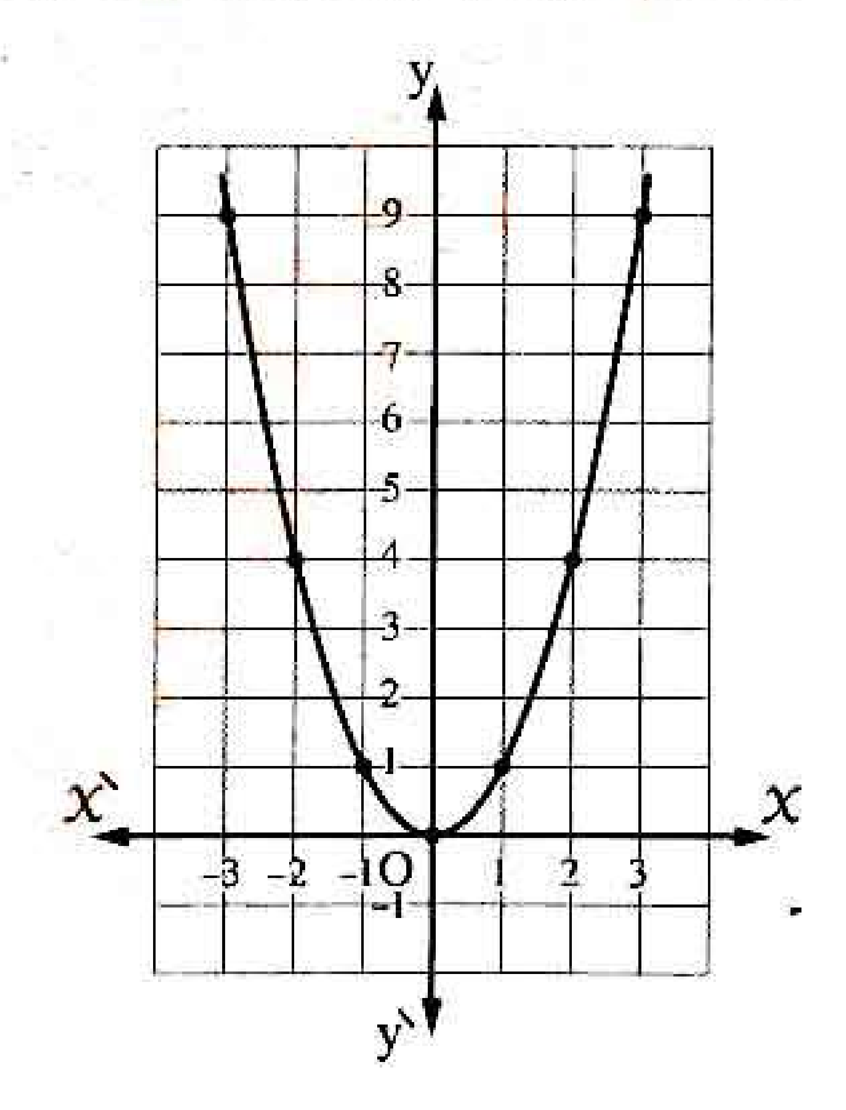
Notice that :

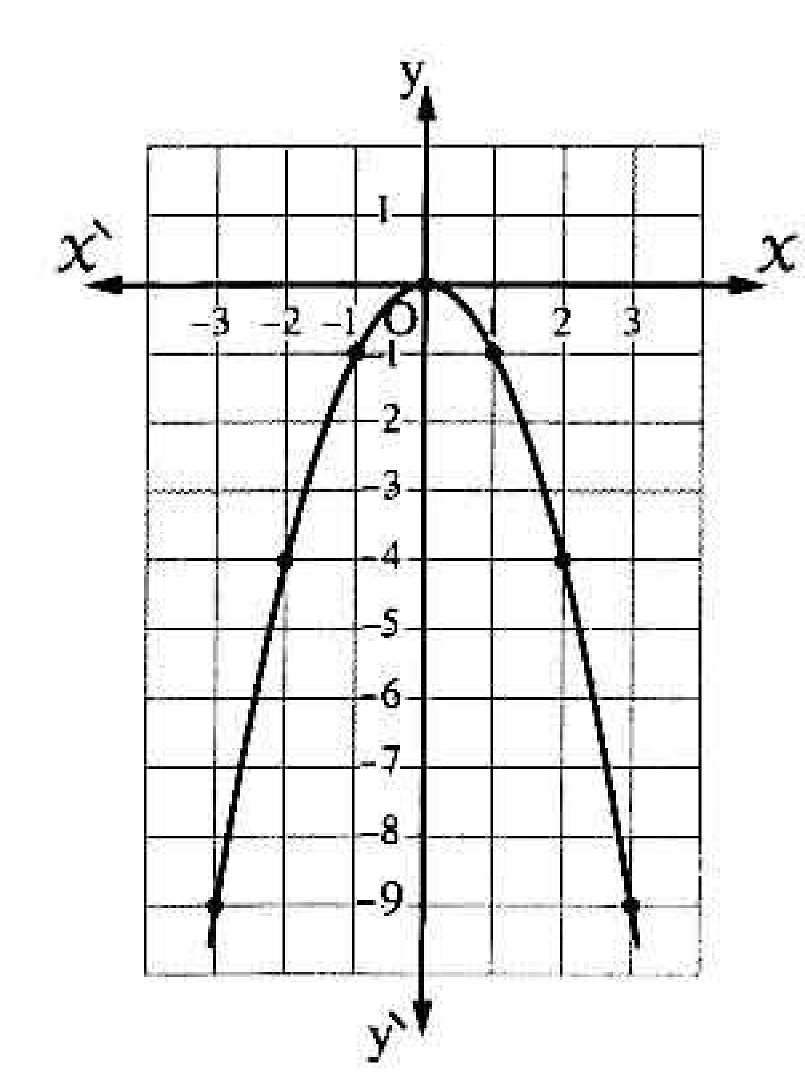
In each of the shown functions , the highest index of X is 2 therefore each of them is a function of the 2^{nd} degree.

Generally , for any quadratic function

1 If the coefficient of X^2 is positive , then the curve is open upwards and the function has a minimum value point.

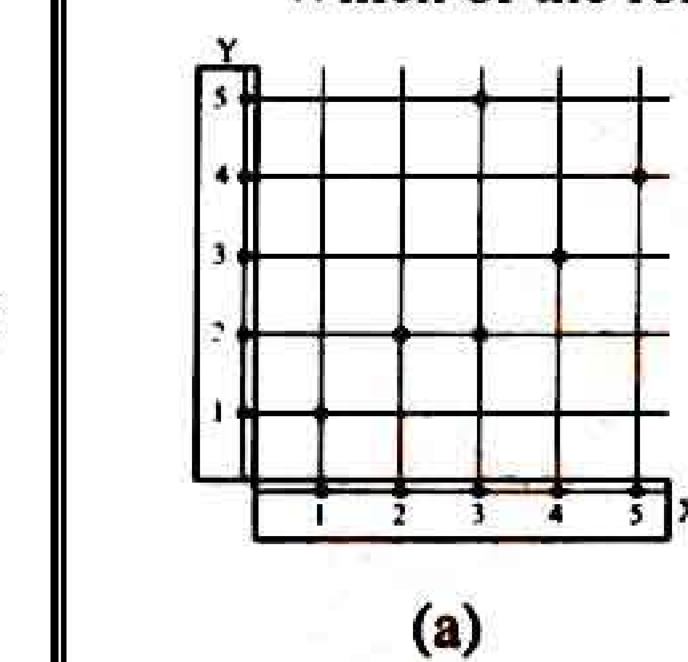
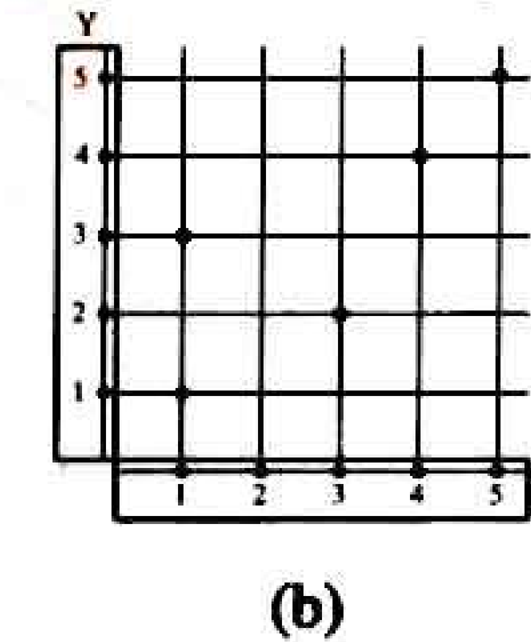
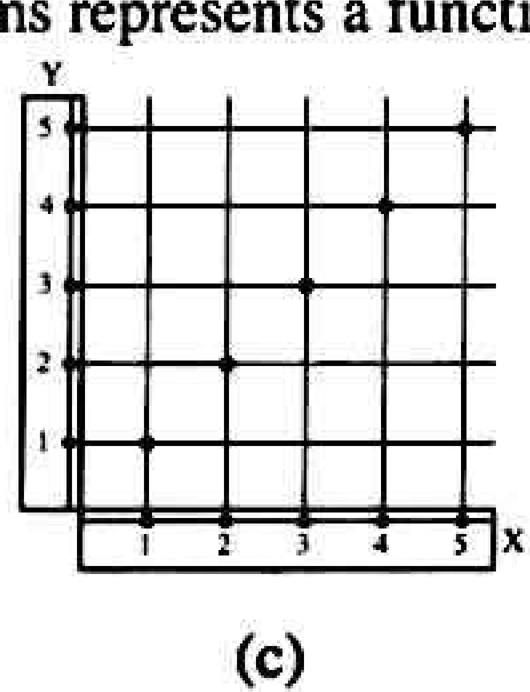
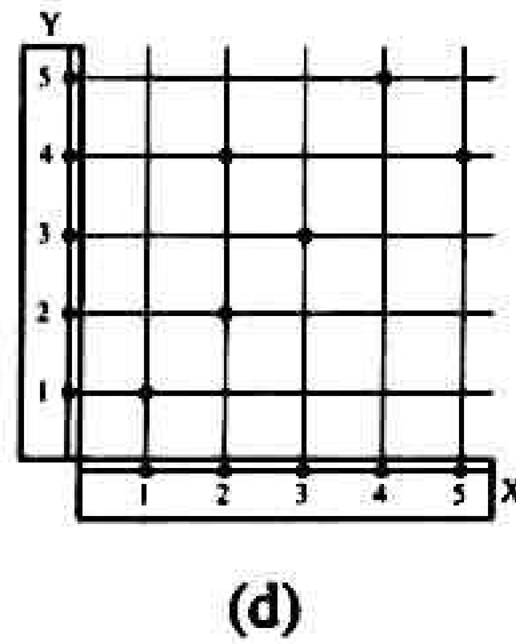
2 If the coefficient of X^2 is negative , then the curve is open downwards and the function has a maximum value point.





Exercises

[A] : Choose The Correct Answer : -

1	If $f(X) = 3$, then $3f(2) - 2f(3) =$ …………… (a) zero (b) 4 (c) 1 (d) 3
2	If the straight line which represents the function $f : f(X) = 2X - a$ passes through the origin point , then a = …………… (a) – 2 (b) 2 (c) 0 (d) 3
3	The ordered pair that satisfies the relation $X + y = 3$ is …………… (a) (1 , – 1) (b) (1 , 2) (c) (– 1 , 1) (d) (0 , 1)
4	If n (X) = 3 , Y = {4 , 5} , then n (X × Y) = …………… (a) 2 (b) 6 (c) 5 (d) 3
5	If n (X) = 2 , n (X × Y) = 8 , then n (Y^2) = …………… (a) 2 (b) 4 (c) 8 (d) 16
6	If $(1, 4) \in \{1, 5\} \times \{X, 7\}$, then $X =$ …………… (a) 1 (b) 2 (c) 3 (d) 4
7	If $(3 - X, X - 1)$ is located in the fourth quadrant where $X \in \mathbb{Z}$, then $X =$ …………… (a) 4 (b) 3 (c) 2 (d) zero
8	If b < 3 , then the point (5 , b – 3) lies in the …………… quadrant. (a) first (b) second (c) third (d) fourth
9	The function f where $f(X) = -2X$ is represented graphically by a straight line passing through the point …………… (a) (– 2 , 0) (b) (0 , – 2) (c) (0 , 0) (d) (– 2 , – 2)
10	Which of the following Cartesian diagrams represents a function from X to Y ?  (a)  (b)  (c)  (d)

11	If n (X) = 2 , $n(Y^2) = 9$, then n (X × Y) = ... (a) 6 (b) 18 (c) 11 (d) 7	
12	If n (X) = 2 , n (Y × X) = 6 , then $n(Y^2)$ = ... (a) 4 (b) 9 (c) 16 (d) 12	
13	If $(3, 5) \in \{3, 6\} \times \{x, 8\}$, then x = ... (a) 8 (b) 5 (c) 6 (d) 3	
14	If the point (5 , b – 7) located on the x-axis, then b = ... (a) 2 (b) 5 (c) 7 (d) 12	
15	If $x \in \mathbb{R}_-$, then the point $\left(-x, \sqrt[3]{x}\right)$ lies in the .... quadrant. (a) first (b) second (c) third (d) fourth	
16	If $f(x) = 2x + b$ and $f(5) = 11$, then b = ... (a) 3 (b) 2 (c) 1 (d) zero	
17	If the relation $R = \{(4, 3), (1, 3), (2, 5)\}$, then R represents a function when its range is .... (a) $\{4, 1, 2\}$ (b) $\{4, 1, 2, 3, 5\}$ (c) $\{3, 5\}$ (d) $\mathbb{N}$	
18	If $X = \{2\}$, $Y = \{0, 4\}$, then n (X × Y) = ... (a) 8 (b) 80 (c) 6 (d) 2	
19	If $X = \{7\}$, then $n(X^2)$ = ... (a) 1 (b) 49 (c) 14 (d) 7	
20	If $(2, 5) \in \{3, 2\} \times \{1, x\}$, then x = ... (a) 2 (b) 3 (c) 1 (d) 5	
21	If $(x + 1, x - 3)$ lies on the x-axis , then x = ... (a) – 1 (b) zero (c) – 2 (d) 3	
22	The point (– 3 , 4) lies in .... quadrant. (a) first (b) second (c) third (d) fourth	
23	If $f(x) = 7$, then $f(-3)$ = ... (a) 7 (b) – 7 (c) 21 (d) – 21	
24	If the function $f : X \longrightarrow Y$, then the range of the function $f \subset$ .... (a) X × Y (b) X (c) Y × X (d) Y	

25	If $X = \{2\}$, $Y = \{1, 3\}$, then $n(X \times Y) =$ .... (a) 1 (b) 4 (c) 3 (d) 2
26	If $X = \{2, 3, 4\}$, then $n(X^2) =$ .... (a) 3 (b) 6 (c) 9 (d) 12
27	If $X = \{3\}$, then $X^2 =$ .... (a) $\{3, 3\}$ (b) $\{(3, 3)\}$ (c) $\{9\}$ (d) $(3, 3)$
28	If the point $(x - 3, 2)$ lies on the y-axis , then $x =$ .... (a) 5 (b) 3 (c) -3 (d) 0
29	The point $(-3, 4)$ lies in .... quadrant. (a) first (b) second (c) third (d) fourth
30	If $f(x) = 5$ is represented by a straight line parallel to the x-axis, then this line passes through the point .... (a) $(0, 5)$ (b) $(5, 0)$ (c) $(5, -5)$ (d) $(0, 0)$
31	The set of images of the elements of the domain of the function is called .... (a) the rule. (b) the domain. (c) the range. (d) the codomain.
32	If $X = \{5\}$, $Y = \{3\}$, then $n(X \times Y) =$ .... (a) 15 (b) 8 (c) 2 (d) 1
33	If $X \times Y = \{(2, 3), (2, 4)\}$, then $n(X) =$ .... (a) 2 (b) 1 (c) 4 (d) 3
34	If $X = \{1\}$, then $X^2 =$ .... (a) 1 (b) $(1, 1)$ (c) $\{(1, 1)\}$ (d) $\{1\}$
35	If the point $(x, 7)$ lies on y-axis , then $5x + 1 =$ .... (a) zero (b) 1 (c) 5 (d) 6
36	The point $(-2, 4)$ lies on the .... quadrant. (a) first (b) second (c) third (d) fourth
37	If $f(x) = 5$, then $f(3) - f(-3) =$ .... (a) zero (b) 10 (c) 6 (d) -6
38	If $f(x) = 4x + b$, $f(3) = 15$, then $b =$ .... (a) 6 (b) 3 (c) 4 (d) -3

39	If $(3 , 5) \in \{(3 , X) , (3 , 8) , (6 , 8)\}$, then $X =$ …………… (a) 8 (b) 6 (c) 5 (d) 3	
40	If $n(X^2) = 4$, $n(X \times Y) = 6$, then $n(Y) =$ …………… (a) 2 (b) 3 (c) 4 (d) 6	
41	If $X = \{2\}$, $Y = \{3\}$, then $X \times Y =$ …………… (a) 6 (b) $\{6\}$ (c) $(2 , 3)$ (d) $\{(2 , 3)\}$	
42	If $(5 , X - 7) = (y + 1 , - 5)$, then $X + y =$ …………… (a) 5 (b) – 1 (c) 6 (d) zero	
43	The point $(- 2 , - 3)$ lies on the …………… quadrant. (a) first (b) second (c) third (d) fourth	
44	If f is a function such that $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 3$, then $\frac{f(6)}{f(\text{zero})} =$ …………… (a) 6 (b) 1 (c) 3 (d) undefined.	
45	If the point $(2 , a - 1)$ lies on the straight line represented to the function $f : f(X) = 4X - 5$, then $a =$ …………… (a) 4 (b) 1 (c) 3 (d) 2	
46	If A , B are two sets , then the set $\{(X , y) : X \in A , y \in B\}$ expresses …………… (a) $n(A \times B)$ (b) $A \times B$ (c) $n(B \times A)$ (d) $B \times A$	
47	If $n(X) = 5$, $n(X \times Y) = 15$, then $n(Y) =$ …………… (a) 3 (b) 5 (c) 15 (d) 8	
48	If $X \times Y = \{(1 , 2) , (1 , 3) , (1 , 4)\}$, then $n(X) + n(Y^2)$ …………… (a) 3 (b) 4 (c) 6 (d) 10	
49	If $(X - 1 , 3) = (1 , y + X)$, then $y =$ …………… (a) 1 (b) – 1 (c) 2 (d) – 2	
50	If the point $(X - 5 , 7 - X)$ lies in the second quadrant, then $X =$ …………… (a) 9 (b) 3 (c) 7 (d) 5	
51	The function $f : f(X) = 3X$ is represented graphically by a straight line passing through the point …………… (a) $(3 , 3)$ (b) $(3 , 0)$ (c) $(0 , 0)$ (d) $(0 , 3)$	

52 If $X = \{1, 2\}$, then the arrow diagram represents a function on X is ....

(a) (b) (c) (d)

53 If n (X) = 5 , n (X × Y) = 10 , then n (Y) = ...

(a) 4 (b) 3 (c) 2 (d) 1

54 If $n(X^2) = 9$, $n(X \times Y) = 6$, then $n(Y^2) =$ ....

(a) 3 (b) 2 (c) 9 (d) 4

55 If $(2^x, 27) = (32, y^3)$, then $\frac{x}{y} =$ ....

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{32}{27}$ (d) $\frac{27}{32}$

56 If the point $(x - 4, 2 - x)$ lies on the fourth quadrant, where $x \in \mathbb{Z}$, then $x =$ ....

(a) 2 (b) 3 (c) 4 (d) 5

57 If the point (a , 5) lies on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = 3x - 4$, then a = ...

(a) 3 (b) – 3 (c) 1 (d) – 1

58 If $X = \{2, 5\}$, which of the following arrow diagrams represents a function on the set X ?

(a) (b) (c) (d)

59 If n (X) = 3 , n (X × Y) = 12 , then n (Y) = ...

(a) 4 (b) 9 (c) 15 (d) 36

60 If n (X × Y) = 6 , n (Y) = 2 , then $n(X^2) =$ ....

16 (b) 9 (c) 4 (d) 1

61 If $(x, 8) = (2, x + y)$, then y = ...

(a) 2 (b) 6 (c) 8 (d) 10

62 If the point $(x - 3, 2 - x)$ lies in the fourth quadrant , then $x =$ ....

(a) 4 (b) 3 (c) 2 1

[C] : Essay Problems : -

1

Represent graphically the linear function $f : f(X) = 2 - X$ and find the point of intersection of the straight line by y-axis.

2017 Exam (18) Question (4) (a)

2

If $f : X \longrightarrow \mathbb{R}$ where $X = \{1 , 2 , 3\}$, $\mathbb{R}$ is the set of real numbers and $f(X) = 2X + 1$

(1) Find the range of the function f

(2) Represent the function f graphically.

2017 Exam (1) Question (5) (a)

3

Draw the curve of the function $f : f(X) = X^2$ on the interval $[-3 , 3]$

and from the graph , find :

(1) The coordinates of the vertex of the curve.

(2) The equation of the axis of symmetry.

(3) The minimum value of f

2017 Exam (10) Question (4) (a)

4

Represent graphically the curve of the function f where :

$f(X) = 1 - X^2$ consider $X \in [-3 , 3]$ **and form the graph find :**

(1) The coordinates of the vertex of the curve.

(2) The equation of axis of symmetry of the function.

2018 Exam (9) Question (5) (b)

5

Draw the curve of the function f such that $f(X) = X^2 - 2X - 3$ taking $X \in [-2 , 4]$,

and from the graph , deduce :

(1) The equation of the axis of symmetry.

(2) The maximum or minimum value of the curve.

2017 Exam (16) Question (5) (b)

6

Graph the function f where $f(X) = X^2 - 4$, $X \in [-3 , 3]$ **and from the graph find :**

(1) The coordinates of the vertex.

(2) The equation of the symmetric axis.

(3) The maximum or the minimum value of the function.

2017 Exam (4) Question (4) (a)

7

Represent graphically the function $f(X) = 4 - X^2$, $X \in [-3 , 3]$, from the graph deduce the vertex of the curve , the maximum value of the function and the equation of the axis of symmetry.

Model Exam (2) Question (5) (a)

8

Graph the curve of the function f , where : $f(x) = (x - 1)^2$ in the interval $[-1 , 3]$

, from the graph determine :

(1) The minimum value of the function.

(2) The equation of the axis of symmetry.

(3) The coordinates of the vertex of the curve.

2018 Exam (1) Question (5) (b)

9

Draw the function curve f where $f(x) = 2x - x^2$ in the interval $[-2 , 4]$

and from the drawing determine :

(1) The coordinates of the vertex point of the curve.

(2) The equation of the symmetry axis.

(3) The maximum or minimum value of the function.

2018 Exam (7) Question (5) (a)

10

The opposite figure represents the function f where $f(x) = 4 - 2x$

Find :

(1) The coordinates of A , B

(2) The area of Δ AOB

2017 Exam (9) Question (3) (a)

11

Represent graphically function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x + 3$

2018 Exam (23) Question (5) (a)

12

Represent graphically the linear function $f : f(x) = 2x + 1$ and find the points of intersection of the straight line with the two coordinate axes.

2018 Exam (11) Question (5) (a)

13

The opposite figure represents the curve of the function f where $f(x) = x^2 + k$

If OA = 9 units.

Find : (1) The value of k

(2) The coordinates of B and C

(3) The area of triangle with vertices A , B and C

2018 Exam (6) Question (4) (a)

Prep. [3]

First Term

Algebra

Unit [2]

Lesson [1]

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Prep. [3] - First Term – Algebra – Unit [2] : Unit [2] : Ratio , Proportion , And Variation

Lesson [1] : Ratio And Proportion

Generally

If a and b are two real numbers , then :

The ratio between a and b is written a : b or $\frac{a}{b}$ and is read a to b where :

a is called the antecedent of the ratio , b is called the consequent of the ratio, a and b are called together the two terms of the ratio.

Properties of the ratio

1 The value of the ratio does not change if each of its terms is multiplied or divided by the same non-zero real number.

2 The value of the ratio ($\neq 1$) changes if we add or subtract (to or from) each of its two terms a non-zero real number.

First : The Proportion: -

Definition of proportion

It is the equality of two ratios or more.

i.e. If $\frac{a}{b} = \frac{c}{d}$, then the quantities a , b , c and d are proportional.

And vice versa : If a , b , c and d are proportional , then : $\frac{a}{b} = \frac{c}{d}$

- a is called the first proportional.
- b is called the second proportional.
- c is called the third proportional.
- d is called the fourth proportional.

a and d are called extremes and b and c are called means.

For example:

The numbers 1 , 4 , 7 and 28 are proportional numbers , because $\frac{1}{4} = \frac{7}{28}$

And : 1 is the first proportional, 4 is the second proportional , 7 is the third proportional , 28 is the fourth proportional, 1 and 28 are the extremes of this proportion and 4 and 7 are the means.

Properties of proportion

Property (1)

If $\frac{a}{b} = \frac{c}{d}$, then : $a \times d = b \times c$ (The product of the extremes = the product of the means)

Property (2)

If $a \times d = b \times c$, then $\frac{a}{b} = \frac{c}{d}$

Also we can deduce that :

- If $a \times d = b \times c$, then $\frac{a}{c} = \frac{b}{d}$
- If $a \times d = b \times c$, then $\frac{b}{a} = \frac{d}{c}$
- If $a \times d = b \times c$, then $\frac{c}{a} = \frac{d}{b}$

Property (3)

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

i.e. $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of the first ratio}}{\text{The consequent of the second ratio}}$

For example:

If $\frac{a}{4} = \frac{b}{3}$, then $\frac{a}{b} = \frac{4}{3}$ and $\frac{b}{a} = \frac{3}{4}$

Property (4)

If $\frac{a}{b} = \frac{c}{d}$, then $a = cm$ and $b = dm$ (where m is a constant $\neq 0$)

For example:

If $\frac{a}{b} = \frac{3}{4}$, then : $a = 3\ m$, $b = 4\ m$ (where m is a constant $\neq 0$)

Important remark

If a , b , c and d are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = m$, then

ⓐ $= bm$, ⓒ $= dm$

For example:

If $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $a = \frac{3}{4}\ b$, $c = \frac{3}{4}\ d$

Generally

If a , b , c , d , e , f , ... are proportional quantities and we assume that :

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = m$, then ⓐ = bm , ⓒ = dm , ⓔ = fm , ···

Property (5)

If we consider the proportion : $\frac{9}{15} = \frac{6}{10} = \frac{3}{5}$

- If we add the antecedents and consequents of the 1st and the 2nd ratios, we get the ratio $\frac{9+6}{15+10} = \frac{15}{25} = \frac{3}{5}$ which is one of given ratios.
- Also if we add the antecedents and consequents of the 2nd and the 3rd ratios, we get the ratio $\frac{6+3}{10+5} = \frac{9}{15} = \frac{3}{5}$ = one of the given ratios.
- If we add the antecedents and consequents of the three given ratios, we get the ratio $\frac{9+6+3}{15+10+5} = \frac{18}{30} = \frac{3}{5}$ = one of the given ratios.

i.e. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$ and m_1 , m_2 , m_3 , ··· are non-zero real numbers

, then $\frac{m_1 a + m_2 c + m_3 e + \cdots}{m_1 b + m_2 d + m_3 f + \cdots}$ = one of the given ratios

Exercises

[A] : Choose The Correct Answer : -

1	The first proportion of the quantities ··· , 21 , 15 , 35 is .... (a) 3 (b) 5 (c) 7 (d) 9
2	If 2 , 3 , 6 and x are proportional , then x = ... (a) 9 (b) 18 (c) 12 (d) 3
3	The fourth proportional to the numbers 3 , 6 , 8 is .... (a) 4 (b) 7 (c) 16 (d) 20
4	The fourth proportional of the quantities 3 , 6 , 6 is .... (a) 3 (b) 6 (c) 12 (d) 9

5	The fourth proportional to the quantities 3 , 9 and 9 is .... (a) 6 (b) 12 (c) 18 (d) 27
6	The fourth proportional for quantities 6 , 21 , 10 is .... (a) 25 (b) 35 (c) 15 (d) 45
7	If a , x , b , 2 x are proportional quantities, then $\frac{a}{b}$ = ... (a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4
8	If a , b , 2 , 3 are proportional , then $\frac{b}{a}$ = ... (a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) 3 (d) 2
9	If a , 2 x , b , 3 x are proportional quantities , then a : b = ... (a) 2 : 1 (b) 3 : 1 (c) 2 : 3 (d) 3 : 2
10	If a , 3 x , b and 5 x are proportional quantities , then $\frac{a}{b}$ = ... (a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{8}{3}$ (d) 15
11	If a , 4 , b and 9 are proportional , then $\frac{a}{b}$ = ... (a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) $\frac{-9}{4}$ (d) $\frac{-4}{9}$
12	If 2 a = 5 b , then $\frac{a}{b}$ = ... (a) $\frac{-5}{2}$ (b) $\frac{-2}{5}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$
13	If 3 a = 4 b , then a : b = ... (a) 3 : 4 (b) 4 : 3 (c) 3 : 7 (d) 4 : 7
14	If 3 a = 5 b , then $\frac{3a}{b}$ = ... (a) 3 (b) 5 (c) $\frac{3}{5}$ (d) $\frac{5}{3}$
15	If $\frac{a}{b} = \frac{5}{2}$, then $\frac{2a}{b}$ = ... (a) 2 (b) 5 (c) 3 (d) 4
16	If $\frac{A}{B} = \frac{5}{3}$, then $\frac{3A}{5B}$ equals .... (a) 1 (b) $\frac{5}{3}$ (c) 3 (d) 15
17	If $\frac{a}{b} = \frac{3}{5}$, then $\frac{5a}{3b}$ = ... (a) $\frac{3}{5}$ (b) 3 (c) 15 (d) 1

18	If $\frac{a}{b} = \frac{c}{d} = m$ where $m \neq 0$, then $\frac{a \times c}{b \times d} =$ …………… (a) $2\ m^2$ (b) m^2 (c) m (d) 2 m
19	If $\frac{x}{5} = \frac{y}{4} = \frac{x + y}{k}$, then k = …………… (a) 20 (b) 9 (c) 1 (d) 45
20	If $\frac{a}{5} = \frac{b}{3} = \frac{c}{4} = \frac{a + b - c}{x}$, then $x =$ …………… (a) 3 (b) 4 (c) 5 (d) 6
21	If $\frac{x}{5} = \frac{y}{4} = \frac{x + 2y}{k}$, then k = …………… (a) 9 (b) 13 (c) 14 (d) 8
22	If $\frac{x}{2} = \frac{y}{3} = \frac{2x + 3y}{k}$, then k = …………… (a) 5 (b) 3 (c) 13 (d) 6
23	If $\frac{x}{2} = \frac{y}{3} = \frac{4x - 2y}{z}$, then z = …………… (a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2
24	If $\frac{9}{a^2} = \frac{4}{b^2}$ (where a and $b \neq 0$) , then $\frac{a}{b} =$ …………… (a) $\frac{2}{3}$ (b) $\pm \frac{3}{2}$ (c) $\pm \frac{2}{3}$ (d) $\pm \frac{4}{9}$
25	If $4x^2 = 9y^2$, then $\frac{x}{y} =$ …………… (a) $\frac{9}{4}$ (b) $\frac{3}{2}$ (c) $\pm \frac{2}{3}$ (d) $\pm \frac{3}{2}$
26	If $\frac{a}{b} = \frac{2}{3}$, $\frac{a}{c} = \frac{4}{5}$, then b : c = …………… (a) 3 : 4 (b) 5 : 6 (c) 6 : 5 (d) 4 : 3
27	If $x \in \mathbb{R}_-$, then the point $(-x , \sqrt[3]{x})$ lies in the …………… quadrant. (a) first (b) second (c) third (d) fourth
28	If the point (5 , b – 7) located on the x-axis, then b = …………… (a) 2 (b) 5 (c) 7 (d) 12
29	If $(3 , 5) \in \{3 , 6\} \times \{x , 8\}$, then $x =$ …………… (a) 8 (b) 5 (c) 6 (d) 3
30	If n (X) = 5 , n (X × Y) = 10 , then n (Y) = …………… (a) 4 (b) 3 (c) 2 (d) 1

31	If $f(\mathcal{X}) = 3$, then $3f(2) - 2f(3) =$ …………… (a) zero (b) 4 (c) 1 (d) 3
32	If $(3 - \mathcal{X} , \mathcal{X} - 1)$ is located in the fourth quadrant where $\mathcal{X} \in \mathbb{Z}$, then $\mathcal{X} =$ …………… (a) 4 (b) 3 (c) 2 (d) zero
33	If $(1 , 4) \in \{1 , 5\} \times \{\mathcal{X} , 7\}$, then $\mathcal{X} =$ …………… (a) 1 (b) 2 (c) 3 (d) 4
34	If $n(X) = 2$, $n(X \times Y) = 8$, then $n(Y^2) =$ …………… (a) 2 (b) 4 (c) 8 (d) 16
35	If A , B are two sets , then the set $\{(\mathcal{X} , y) : \mathcal{X} \in A , y \in B\}$ expresses …………… (a) $n(A \times B)$ (b) $A \times B$ (c) $n(B \times A)$ (d) $B \times A$
36	If $(3 , 5) \in \{(3 , \mathcal{X}) , (3 , 8) , (6 , 8)\}$, then $\mathcal{X} =$ …………… (a) 8 (b) 6 (c) 5 (d) 3
37	If $f(\mathcal{X}) = n\mathcal{X}^2 + 2\mathcal{X}^n - 3$, then the possible set of values of n such that f is a function of the second degree is …………… (a) $\{2 , 3\}$ (b) $\{1 , -1\}$ (c) $\{2 , 1 , 0\}$ (d) $\{2 , 1\}$
38	The function $f : f(\mathcal{X}) = 3\mathcal{X}$ is represented graphically by a straight line passing through the point …………… (a) (3 , 3) (b) (3 , 0) (c) (0 , 0) (d) (0 , 3)
39	If the point $(\mathcal{X} - 4 , 2 - \mathcal{X})$ lies on the fourth quadrant, where $\mathcal{X} \in \mathbb{Z}$, then $\mathcal{X} =$ …………… (a) 2 (b) 3 (c) 4 (d) 5
40	If f is a function such that $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(\mathcal{X}) = 3$, then $\frac{f(6)}{f(\text{zero})} =$ …………… (a) 6 (b) 1 (c) 3 (d) undefined.
41	If the point $(\mathcal{X} - 5 , 7 - \mathcal{X})$ lies in the second quadrant, then $\mathcal{X} =$ …………… (a) 9 (b) 3 (c) 7 (d) 5
42	If $(\mathcal{X} - 1 , 3) = (1 , y + \mathcal{X})$, then $y =$ …………… (a) 1 (b) – 1 (c) 2 (d) – 2
43	If $X \times Y = \{(1 , 2) , (1 , 3) , (1 , 4)\}$, then $n(X) + n(Y^2)$ …………… (a) 3 (b) 4 (c) 6 (d) 10
44	If $(5 , \mathcal{X} - 7) = (y + 1 , -5)$, then $\mathcal{X} + y =$ …………… (a) 5 (b) – 1 (c) 6 (d) zero

45	If $X = \{2\}$, $Y = \{3\}$, then $X \times Y =$ .... (a) 6 (b) $\{6\}$ (c) $(2, 3)$ (d) $\{(2, 3)\}$
46	If $X = \{2\}$, $Y = \{0, 4\}$, then $n(X \times Y) =$ .... (a) 8 (b) 80 (c) 6 (d) 2
47	If the function $f : X \longrightarrow Y$, then the range of the function $f \subset$ .... (a) $X \times Y$ (b) X (c) $Y \times X$ (d) Y
48	If $n(X) = 2$, $n(Y^2) = 9$, then $n(X \times Y) =$ .... (a) 6 (b) 18 (c) 11 (d) 7
49	If the relation $R = \{(4, 3), (1, 3), (2, 5)\}$, then R represents a function when its range is .... (a) $\{4, 1, 2\}$ (b) $\{4, 1, 2, 3, 5\}$ (c) $\{3, 5\}$ (d) $\mathbb{N}$
50	The function f where $f(X) = X^2 - (X^2 - 3X)$ is polynomial of the .... degree. (a) first (b) second (c) third (d) fourth
51	If $f(X) = 7$, then $f(-3) =$ .... (a) 7 (b) – 7 (c) 21 (d) – 21
52	If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = X^3 - (X^3 - 3)$, then the degree of the function is the .... (a) first (b) second (c) third (d) zero
53	If $f(X) = 2X + b$ and $f(5) = 11$, then $b =$ .... (a) 3 (b) 2 (c) 1 (d) zero
54	The point $(-3, 4)$ lies in .... quadrant. (a) first (b) second (c) third (d) fourth
55	If $(X + 1, X - 3)$ lies on the X-axis , then $X =$ .... (a) – 1 (b) zero (c) – 2 (d) 3

[C] : Essay Problems : -

1	If a , b , c and d are proportional. **Prove that :** $\frac{a}{b-a}=\frac{c}{d-c}$ Model Exam (1) Question (2) (b)
2	If a , b , c and d are proportional , **then prove that :** $\frac{2a+3b}{2c+3d}=\frac{a}{c}$ 2017 Exam (14) Question (2) (a)
3	If a , b , c and d are proportional quantities, **then prove that :** $\frac{3a-6c}{b-2d}=\frac{3a}{b}$ 2018 Exam (7) Question (4) (b)
4	If x , y , r and m are proportional quantities , **then prove that :** $\frac{3x+2r}{3y+2m}=\frac{x-r}{y-m}$ 2017 Exam (5) Question (3) (b)
5	If $\frac{x}{y}=\frac{2}{3}$, **find the value of :** $\frac{3x-y}{2x+y}$ 2018 Exam (21) Question (5) (a)
6	If $3x=2y$, **find the value of the ratio :** $\frac{2x+2y}{6y-x}$ 2017 Exam (20) Question (4) (a)
7	If $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}$, **prove that :** $\frac{a+b-c}{a-b+c}=\frac{1}{3}$ 2018 Exam (5) Question (2) (a)
8	If $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$, **then find the value of :** $\frac{2y-z}{3x-2y+z}$ 2018 Exam (2) Question (3) (a)
9	If $\frac{x}{5}=\frac{y}{3}=\frac{z}{6}$, **prove that :** $\frac{2x+y-z}{7}=\frac{y+z}{9}$ 2017 Exam (8) Question (3) (a)
10	If a : b : c = 4 : 5 : 3 , **then prove that :** $\frac{a-b+c}{a+b-c}=\frac{1}{3}$ 2017 Exam (13) Question (1) (b)
11	If $\frac{x+2y}{3x-2y}=\frac{3}{2}$, **then find the value of :** $\frac{x}{y}$ 2017 Exam (5) Question (5) (a)
12	If $\frac{x}{2a+b}=\frac{y}{2b-c}=\frac{z}{2c-a}$, **then prove that :** $\frac{2x+y}{4a+4b-c}=\frac{2x+2y+z}{3a+6b}$ 2018 Exam (14) Question (2) (b)

13	If $\frac{a+b}{5}=\frac{b+c}{3}=\frac{c+a}{6}$, **prove that :** $\frac{a+b+c}{a-c}=\frac{7}{2}$ 2018 Exam (8) Question (4) (b)
14	Find the positive number if we add its square to each term of the ratio 5 : 11 it becomes 3 : 5 2018 Exam (13) Question (3) (a)
15	Find the number which if its square is added to each of the two terms of the ratio 7 : 11 it becomes 4 : 5 2018 Exam (22) Question (4) (b)
16	Two integers , the ratio between them is 2 : 3 , if you add to the first 7 and subtract from the second 12 , the ratio between them becomes 5 : 3 , find the two numbers. 2018 Exam (16) Question (2) (b)
17	If a , b , c and d are proportional quantities , **prove that :** $\frac{a+2c}{b+2d}=\frac{c-a}{d-b}$ 2017 Exam (1) Question (4) (a)
18	If a , b , c and d are proportion quantities , **prove that :** $\frac{3a+c}{5a-2c}=\frac{3b+d}{5b-2d}$ 2018 Exam (18) Question (4) (b)
19	If a , b , c and d are proportional quantities , **prove that :** $\frac{a^2+c^2}{b^2+d^2}=\frac{ac}{bd}$ 2018 Exam (4) Question (4) (a)
20	If a , b , c and d are four real proportional quantities , **prove that :** $\left(\frac{a+2c}{b+2d}\right)^2=\frac{ac}{bd}$ 2018 Exam (24) Question (3) (b)
21	If 5 a = 3 b **Find the value of :** $\frac{7a+9b}{4a+2b}$ Model Exam (2) Question (3) (b)
22	If $\frac{a}{b}=\frac{2}{5}$, **find the numerical value of :** $\frac{b-a}{b+a}$ 2017 Exam (1) Question (2) (b)

Prep. [3]

First Term

Algebra

Unit [2]

Lesson [2]

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Prep. [3] - First Term – Algebra – Unit [2] : Unit [2] : Ratio , Proportion , And Variation

Lesson [2] : Continued Proportion

Definition :

The quantities a , b and c are said to be in continued proportion if $\frac{a}{b} = \frac{b}{c}$

In this proportion , **a** is called the **first proportion** , **c** is called the **third proportion** and **b** is called the **middle proportion (proportional mean)**

$\because \frac{a}{b} = \frac{b}{c}$ $\therefore b^2 = ac$ $\therefore b = \pm\sqrt{ac}$

i.e.

The middle proportion between two quantities = $\pm\sqrt{\text{the product of the two quantities}}$

Notice that :

The two quantities a and c should be either positive together or negative together.

Remarks

For any two positive numbers or any two negative numbers x and y , there are two middle proportions ($\sqrt{x y}$ and $-\sqrt{x y}$)

i.e. If $\frac{a}{b} = \frac{b}{c} = m$, then $\begin{cases} b = cm \\ a = cm^2 \end{cases}$

i.e. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then $c = dm$, $b = dm^2$ and $a = dm^3$

Exercises

[A] : Choose The Correct Answer : -

1	The third proportation of the two numbers 3 and 6 is .... (a) $\frac{1}{2}$ (b) 9 (c) 2 (d) 12
2	the third proportion of the two numbers 4 and 6 is .... (a) 9 (b) 12 (c) 24 (d) $\frac{3}{2}$

3	The middle proportional between 3 , 27 is .... (a) -9 (b) 9 (c) ± 9 (d) 81
4	The positive middle proportional of the two numbers 1 and 16 is .... (a) 8 (b) 16 (c) 4 (d) 1
5	The middle proportional between x and y is .... (a) $\sqrt{x y}$ (b) $-\sqrt{x y}$ (c) $\pm\sqrt{x y}$ (d) x y
6	The middle proportional between $3 a^3 b$, $27 a b^3$ is .... (a) $-9 a^2b^2$ (b) 9 a b (c) $\pm 9 a^2 b^2$ (d) $9 a^2 b^2$
7	If 3, x and 12 are proportional quantities , then $x =$ .... (a) 15 (b) -6 (c) 6 (d) ± 6
8	If 2 , 6 , $x + 15$ are in proportion , then $x =$ .... (a) 1 (b) 2 (c) 3 (d) 4
9	If a , 2 , 4 and b are in continued proportion, then a + b = ... (a) 2 (b) 4 (c) 6 (d) 9
10	The first proportion of the quantities ··· , 21 , 15 , 35 is .... (a) 3 (b) 5 (c) 7 (d) 9
11	If 2 , 3 , 6 and x are proportional , then $x =$ .... (a) 9 (b) 18 (c) 12 (d) 3
12	The fourth proportional to the numbers 3 , 6 , 8 is .... (a) 4 (b) 7 (c) 16 (d) 20
13	The fourth proportional of the quantities 3 , 6 , 6 is .... (a) 3 (b) 6 (c) 12 (d) 9
14	The fourth proportional to the quantities 3 , 9 and 9 is .... (a) 6 (b) 12 (c) 18 (d) 27
15	The fourth proportional for quantities 6 , 21 , 10 is .... (a) 25 (b) 35 (c) 15 (d) 45
16	If a , x , b , 2 x are proportional quantities, then $\frac{a}{b} =$ .... (a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

17	If a , b , 2 , 3 are proportional , then $\frac{b}{a}$ = ... (a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) 3 (d) 2
18	If a , 2 x , b , 3 x are proportional quantities , then a : b = ... (a) 2 : 1 (b) 3 : 1 (c) 2 : 3 (d) 3 : 2
19	If a , 3 x , b and 5 x are proportional quantities , then $\frac{a}{b}$ = ... (a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{8}{3}$ (d) 15
20	If a , 4 , b and 9 are proportional , then $\frac{a}{b}$ = ... (a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) $\frac{-9}{4}$ (d) $\frac{-4}{9}$
21	If 2 a = 5 b , then $\frac{a}{b}$ = ... (a) $\frac{-5}{2}$ (b) $\frac{-2}{5}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$
22	If 3 a = 4 b , then a : b = ... (a) 3 : 4 (b) 4 : 3 (c) 3 : 7 (d) 4 : 7
23	If 3 a = 5 b , then $\frac{3a}{b}$ = ... (a) 3 (b) 5 (c) $\frac{3}{5}$ (d) $\frac{5}{3}$
24	If $\frac{a}{b} = \frac{5}{2}$, then $\frac{2a}{b}$ = ... (a) 2 (b) 5 (c) 3 (d) 4
25	If $\frac{A}{B} = \frac{5}{3}$, then $\frac{3A}{5B}$ equals .... (a) 1 (b) $\frac{5}{3}$ (c) 3 (d) 15
26	If $\frac{a}{b} = \frac{3}{5}$, then $\frac{5a}{3b}$ = ... (a) $\frac{3}{5}$ (b) 3 (c) 15 (d) 1
27	If $\frac{a}{b} = \frac{c}{d}$ = m where m $\neq$ 0 , then $\frac{a \times c}{b \times d}$ = ... (a) 2 m^2 (b) m^2 (c) m (d) 2 m
28	If $\frac{x}{5} = \frac{y}{4} = \frac{x+y}{k}$, then k = ... (a) 20 (b) 9 (c) 1 (d) 45
29	If $\frac{a}{5} = \frac{b}{3} = \frac{c}{4} = \frac{a+b-c}{x}$, then x = ... (a) 3 (b) 4 (c) 5 (d) 6

30 If $\frac{x}{5} = \frac{y}{4} = \frac{x + 2y}{k}$, then k = ...

(a) 9 (b) 13 (c) 14 (d) 8

31 If $\frac{x}{2} = \frac{y}{3} = \frac{2x + 3y}{k}$, then k = ...

(a) 5 (b) 3 (c) 13 (d) 6

32 If $\frac{x}{2} = \frac{y}{3} = \frac{4x - 2y}{z}$, then z = ...

(a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2

33 If $\frac{9}{a^2} = \frac{4}{b^2}$ (where a and b $\neq 0$) , then $\frac{a}{b}$ = ...

(a) $\frac{2}{3}$ (b) $\pm \frac{3}{2}$ (c) $\pm \frac{2}{3}$ (d) $\pm \frac{4}{9}$

34 If $4x^2 = 9y^2$, then $\frac{x}{y}$ = ...

(a) $\frac{9}{4}$ (b) $\frac{3}{2}$ (c) $\pm \frac{2}{3}$ (d) $\pm \frac{3}{2}$

35 If $\frac{a}{b} = \frac{2}{3}$, $\frac{a}{c} = \frac{4}{5}$, then b : c = ...

(a) 3 : 4 (b) 5 : 6 (c) 6 : 5 (d) 4 : 3

36 If $(x , 8) = (2 , x + y)$, then y = ...

(a) 2 (b) 6 (c) 8 (d) 10

37 The function f where $f(x) = -2x$ is represented graphically by a straight line passing through the point ....

(a) (– 2 , 0) (b) (0 , – 2) (c) (0 , 0) (d) (– 2 , – 2)

38 If X = {1 , 3 , 5} , $f : X \longrightarrow \mathbb{R}$ and $f(x) = 2x + 1$, then the set of images of the elements of the domain of the function f is ....

(a) {3 , 5 , 11} (b) {3 , 7 , 9} (c) {1 , 3 , 11} (d) {3 , 11 , 7}

39 If X = {1} , then X^2 = ...

(a) 1 (b) (1 , 1) (c) {(1 , 1)} (d) {1}

40 The point (– 2 , – 3) lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

41 If the point (2 , a – 1) lies on the straight line represented to the function $f : f(x) = 4x - 5$, then a = ...

(a) 4 (b) 1 (c) 3 (d) 2

42 If n (X × Y) = 6 , n (Y) = 2 , then n (X^2) = ...

(a) 16 (b) 9 (c) 4 (d) 1

[C] : Essay Problems : -

1	If b is the middle proportional between a and c , a = 4 c = 4 , **then find the value of :** $a^2 + b^2 + c^2$ 2018 Exam (15) Question (3) (a)
2	If y is the middle proportional between x and z , **prove that :** $\frac{x}{x+y} = \frac{x z}{y^2 + y z}$ 2017 Exam (11) Question (4) (b)
3	If b is a middle proportional between a and c **Prove that :** $\frac{a-b}{a-c} = \frac{b}{b+c}$ Model Exam (2) Question (2) (b)
4	If b is a middle proportion between a and c , **prove that :** $\frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$ 2018 Exam (12) Question (3) (b)
5	If b is a middle proportion between a and c , **prove that :** $\frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a}$ 2018 Exam (14) Question (3) (b)
6	If a , b , c are proportional quantities , **prove that :** $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$ 2017 Exam (19) Question (2) (b)
7	If a , b and c are proportional quantities , **prove that :** $\frac{2a+3b}{2b+3c} = \frac{a}{b}$ 2018 Exam (9) Question (3) (b)
8	If a , b , c and d are in continued proportional. **Prove that :** $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$ 2018 Exam (6) Question (5) (b)
9	Find the number that if we add it to each of the numbers : 1 , 5 , 17 , then they become in continued proportion. 2018 Exam (19) Question (3) (b)
10	If y is the middle proportional between x and z , **then prove that :** $\frac{x z}{y^2 + y z} = \frac{x}{x+y}$ 2018 Exam (13) Question (5) (b)
11	If b is the middle proportional between a and c , **then prove that :** $\frac{a}{c} = \frac{a^2+b^2}{b^2+c^2}$ 2018 Exam (1) Question (5) (a)

Prep. [3]

First Term

Algebra

Unit [2]

Lesson [3]

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Prep. [3] - First Term – Algebra – Unit [2] : Unit [2] : Ratio , Proportion , And Variation

Lesson [3] : Direct Variation And Inverse Variation

First The direct variation

Definition

It is said that y varies directly as x and it is written $y \propto x$ if $y = m x$

i.e. $\frac{y}{x} = m$ (where m is a constant $\neq 0$)

If the variable x took the two values x_1 and x_2 and y took the two values y_1 and y_2 respectively , then : $\frac{y_1}{y_2} = \frac{x_1}{x_2}$

Second The inverse variation

Definition

It is said that y varies inversely as x and it is written $y \propto \frac{1}{x}$ if $y = \frac{m}{x}$

i.e. $x y = m$, where (m is a constant $\neq 0$)

If the variable x took the two values x_1 , x_2 and as a result for that y took the two values y_1 and y_2 respectively , then : $\frac{y_1}{y_2} = \frac{x_2}{x_1}$

Exercises

[A] : Choose The Correct Answer : -

1	If $x y = 3$, then $y \propto$ .... (a) $3x$ (b) $\frac{3}{x}$ (c) $\frac{1}{x}$ (d) $\frac{x}{3}$
2	If $x y = 7$, then $y \propto$ .... (a) $\frac{1}{x}$ (b) $x - 7$ (c) x (d) $x + 7$

3	If $3xy = 8$, then .... (a) $x \propto y$ (b) $y \propto x$ (c) $3x \propto 8y$ (d) $x \propto \frac{1}{y}$
4	If $5xy = 7$, then .... (a) $x \propto y$ (b) $x \propto \frac{1}{y}$ (c) $y \propto x$ (d) $5x \propto 7y$
5	If $x^2 y = 5$, then .... (a) $y \propto x$ (b) $y \propto x^2$ (c) $y \propto \frac{1}{x}$ (d) $y \propto \frac{1}{x^2}$
6	If $xy - 7 = 0$, then $y \propto$ .... (a) $\frac{1}{x}$ (b) $\frac{7}{x}$ (c) $7x$ (d) $\frac{x}{7}$
7	If $y = \frac{5}{x}$, then .... (a) $y \propto \frac{1}{x}$ (b) $y \propto x^2$ (c) $y \propto x$ (d) $y \propto \frac{1}{x^2}$
8	If $y = 2x$, then .... (a) $y \propto x$ (b) $y \propto \frac{1}{x}$ (c) $y \propto \frac{1}{x^2}$ (d) $y \propto x + 2$
9	If $y = 5x$, then $y \propto$ .... (a) x (b) $x + 5$ (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$
10	If $x \propto y$, then $x =$ .... , where m is a non-zero constant. (a) $m + y$ (b) $\frac{m}{y}$ (c) $\frac{1}{my}$ (d) my
11	If $y \propto x$ and $y = 2$ when $x = 8$, then $y = 3$ when $x =$ .... (a) 16 (b) 12 (c) 24 (d) 6
12	If y varies inversely as x and $y = 2$ when $x = \frac{1}{2}$, then the constant of variation is .... (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 4
13	If $y \propto x$ and $x = 1$ when $y = 4$, then the constant of proportion equals .... (a) 4 (b) 3 (c) 1 (d) $\frac{1}{4}$
14	If x varies inversely with y , then $\frac{y_1}{y_2} =$ .... (a) $\frac{m x_1}{x_2}$ (b) $\frac{x_1}{x_2}$ (c) $\frac{x_2}{x_1}$ (d) $\frac{1}{x_1 x_2}$
15	The relation representing the direct variation between the two variables x and y is .... (a) $xy = 2$ (b) $y = 2x$ (c) $y = x + 2$ (d) $\frac{x}{3} = \frac{5}{y}$

16	The relation which represents a direct variation between the two variables x and y is .... (a) $xy = 3$ (b) $y = x + 3$ (c) $\frac{x}{3} = \frac{3}{y}$ (d) $\frac{y}{2} = \frac{x}{3}$
17	The relation which represents direct variation between the two variables x and y is .... (a) $xy = 5$ (b) $y = x + 3$ (c) $\frac{x}{3} = \frac{4}{y}$ (d) $\frac{x}{5} = \frac{y}{2}$
18	The relation which represents direct variation between the two variables x and y is .... (a) $xy = 7$ (b) $y = x + 2$ (c) $\frac{x}{3} = \frac{4}{y}$ (d) $\frac{x}{5} = \frac{y}{2}$
19	The relation which represent a varies inversely between x , y is .... (a) $y = x$ (b) $y = x^2$ (c) $xy^2 = 1$ (d) $y = \frac{3}{x}$
20	If 3 , x and $\frac{1}{y}$ are in proportion, then .... = 3 (a) x^2y (b) y (c) xy (d) $\frac{x^2}{y}$
21	If $x^2 - 4xy^2 + 4y^4 = 0$, then $x \propto$ .... (a) y (b) y^2 (c) $\frac{1}{y}$ (d) $\frac{1}{y^2}$
22	If $x^2y^2 + \frac{1}{4} = xy$, then .... (a) $x \propto y$ (b) $y \propto x$ (c) $2x \propto 5y$ (d) $y \propto \frac{1}{x}$
23	If $y^2 - 4xy + 4x^2 = 0$, then .... (a) $y \propto x$ (b) $y \propto x^2$ (c) $y \propto \frac{1}{x^2}$ (d) $y \propto \frac{1}{x}$
24	The graph which represents the direct variation between x and y is the graph no. .... (a) (b) (c) (d)
25	The third proportation of the two numbers 3 and 6 is .... (a) $\frac{1}{2}$ (b) 9 (c) 2 (d) 12
26	the third proportion of the two numbers 4 and 6 is .... (a) 9 (b) 12 (c) 24 (d) $\frac{3}{2}$

27	The middle proportional between 3 , 27 is .... (a) -9 (b) 9 (c) ± 9 (d) 81
28	The positive middle proportional of the two numbers 1 and 16 is .... (a) 8 (b) 16 (c) 4 (d) 1
29	The middle proportional between x and y is .... (a) $\sqrt{xy}$ (b) $-\sqrt{xy}$ (c) $\pm\sqrt{xy}$ (d) xy
30	The middle proportional between $3a^3b$, $27ab^3$ is .... (a) $-9a^2b^2$ (b) $9ab$ (c) $\pm 9a^2b^2$ (d) $9a^2b^2$
31	If 3, x and 12 are proportional quantities , then x = ... (a) 15 (b) -6 (c) 6 (d) ± 6
32	If 2 , 6 , $x + 15$ are in proportion , then x = ... (a) 1 (b) 2 (c) 3 (d) 4
33	If a , 2 , 4 and b are in continued proportion, then a + b = ... (a) 2 (b) 4 (c) 6 (d) 9
34	The first proportion of the quantities ··· , 21 , 15 , 35 is .... (a) 3 (b) 5 (c) 7 (d) 9
35	If 2 , 3 , 6 and x are proportional , then x = ... (a) 9 (b) 18 (c) 12 (d) 3
36	The fourth proportional to the numbers 3 , 6 , 8 is .... (a) 4 (b) 7 (c) 16 (d) 20
37	The fourth proportional of the quantities 3 , 6 , 6 is .... (a) 3 (b) 6 (c) 12 (d) 9
38	The fourth proportional to the quantities 3 , 9 and 9 is .... (a) 6 (b) 12 (c) 18 (d) 27
39	The fourth proportional for quantities 6 , 21 , 10 is .... (a) 25 (b) 35 (c) 15 (d) 45
40	If a , x , b , 2 x are proportional quantities, then $\frac{a}{b}$ = ... (a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

41	If a , b , 2 , 3 are proportional , then $\frac{b}{a}$ = ... (a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) 3 (d) 2
42	If a , 2 x , b , 3 x are proportional quantities , then a : b = ... (a) 2 : 1 (b) 3 : 1 (c) 2 : 3 (d) 3 : 2
43	If a , 3 x , b and 5 x are proportional quantities , then $\frac{a}{b}$ = ... (a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{8}{3}$ (d) 15
44	If a , 4 , b and 9 are proportional , then $\frac{a}{b}$ = ... (a) $\frac{9}{4}$ (b) $\frac{4}{9}$ (c) $\frac{-9}{4}$ (d) $\frac{-4}{9}$
45	If 2 a = 5 b , then $\frac{a}{b}$ = ... (a) $\frac{-5}{2}$ (b) $\frac{-2}{5}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$
46	If 3 a = 4 b , then a : b = ... (a) 3 : 4 (b) 4 : 3 (c) 3 : 7 (d) 4 : 7
47	If 3 a = 5 b , then $\frac{3a}{b}$ = ... (a) 3 (b) 5 (c) $\frac{3}{5}$ (d) $\frac{5}{3}$
48	If $\frac{a}{b} = \frac{5}{2}$, then $\frac{2a}{b}$ = ... (a) 2 (b) 5 (c) 3 (d) 4
49	If $\frac{A}{B} = \frac{5}{3}$, then $\frac{3A}{5B}$ equals .... (a) 1 (b) $\frac{5}{3}$ (c) 3 (d) 15
50	If $\frac{a}{b} = \frac{3}{5}$, then $\frac{5a}{3b}$ = ... (a) $\frac{3}{5}$ (b) 3 (c) 15 (d) 1
51	If $\frac{a}{b} = \frac{c}{d} = m$ where $m \neq 0$, then $\frac{a \times c}{b \times d}$ = ... (a) $2\ m^2$ (b) m^2 (c) m (d) 2 m
52	If $\frac{x}{5} = \frac{y}{4} = \frac{x + y}{k}$, then k = ... (a) 20 (b) 9 (c) 1 (d) 45
53	If $\frac{a}{5} = \frac{b}{3} = \frac{c}{4} = \frac{a + b - c}{x}$, then x = ... (a) 3 (b) 4 (c) 5 (d) 6

54	If $\frac{x}{5} = \frac{y}{4} = \frac{x + 2y}{k}$, then k = ... (a) 9 (b) 13 (c) 14 (d) 8
55	If $\frac{x}{2} = \frac{y}{3} = \frac{2x + 3y}{k}$, then k = ... (a) 5 (b) 3 (c) 13 (d) 6
56	If $\frac{x}{2} = \frac{y}{3} = \frac{4x - 2y}{z}$, then z = ... (a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2
57	If $\frac{9}{a^2} = \frac{4}{b^2}$ (where a and b $\neq 0$) , then $\frac{a}{b}$ = ... (a) $\frac{2}{3}$ (b) $\pm\frac{3}{2}$ (c) $\pm\frac{2}{3}$ (d) $\pm\frac{4}{9}$
58	If $4x^2 = 9y^2$, then $\frac{x}{y}$ = ... (a) $\frac{9}{4}$ (b) $\frac{3}{2}$ (c) $\pm\frac{2}{3}$ (d) $\pm\frac{3}{2}$
59	If $\frac{a}{b} = \frac{2}{3}$, $\frac{a}{c} = \frac{4}{5}$, then b : c = ... (a) 3 : 4 (b) 5 : 6 (c) 6 : 5 (d) 4 : 3
60	If $n(X^2) = 9$, $n(X \times Y) = 6$, then $n(Y^2)$ = ... (a) 3 (b) 2 (c) 9 (d) 4
61	If the point (a , 5) lies on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = 3x - 4$, then a = ... (a) 3 (b) -3 (c) 1 (d) -1
62	If the straight line which represents the function $f : f(x) = 2x - a$ passes through the origin point , then a = ... (a) -2 (b) 2 (c) 0 (d) 3
63	The ordered pair that satisfies the relation $x + y = 3$ is .... (a) (1 , -1) (b) (1 , 2) (c) (-1 , 1) (d) (0 , 1)
64	If $X = \{3\}$, then X^2 = ... (a) $\{3 , 3\}$ (b) $\{(3 , 3)\}$ (c) $\{9\}$ (d) (3 , 3)
65	The point (-2 , 4) lies on the .... quadrant. (a) first (b) second (c) third (d) fourth
66	If $f(x) = 4x + b$, $f(3) = 15$, then b = ... (a) 6 (b) 3 (c) 4 (d) -3

67	If $X = \{5\}$, $Y = \{3\}$, then $n(X \times Y) =$ .... (a) 15 (b) 8 (c) 2 (d) 1
68	If the point $(X - 3 , 2 - X)$ lies in the fourth quadrant , then $X =$ .... (a) 4 (b) 3 (c) 2 (d) 1
69	If $f(X) = X^3$, then $f(1) + f(-1) =$ .... (a) 2 (b) – 2 (c) zero (d) 4
70	If $n(X) = 3$, $Y = \{4 , 5\}$, then $n(X \times Y) =$ .... (a) 2 (b) 6 (c) 5 (d) 3
71	If the point $(X , 7)$ lies on y-axis , then $5X + 1 =$ .... **(a)** zero **(b)** 1 **(c)** 5 **(d)** 6
72	If $f(X) = 5$, then $f(3) - f(-3) =$ .... (a) zero (b) 10 (c) 6 (d) – 6
73	The function $f : \mathbb{R} \longrightarrow \mathbb{R} : f(X) = a^2 X + a$, where $a \neq 0$ is a polynomial of the .... degree. (a) first (b) second (c) third (d) fourth

[C] : Essay Problems : -

1	If $y \propto X$ and $y = 2$ when $X = 1$, find the value of X when $y = 10$ 2017 Exam (14) Question (3) (b)
2	If $y \propto X$ and $y = 3$ when $X = 10$, **find :** (1) The relation between X and y (2) The value of y when $X = 5$ 2017 Exam (10) Question (5) (a)
3	If $y \propto X$, $y = 6$ when $X = 3$ **Find :** (1) The relation between X and y (2) The value of y when $X = 5$ Model Exam (2) Question (4) (b)
4	If y varies inversely as X and $y = 3$ when $X = 2$, find the value of X when $y = 18$ 2017 Exam (17) Question (5) (b)

5	If $y \propto \frac{1}{x}$ and y = 5 , when $x = 2$ **Find :** (1) The relation between y and x (2) The value of y when $x = 4$ 2018 Exam (4) Question (2) (b)
6	If $y \propto \frac{1}{x}$ and y = 10 when $x = 3$, then find the value of y when $x = 5$ 2018 Exam (23) Question (3) (a)
7	If y varies inversely with x and y = 21 when $x = 4$, then find the value of : y when $x = 7$ 2017 Exam (15) Question (2) (b)
8	If y = 1 + a where a varies inversely as the square of x and y = 17 , when $x = \frac{1}{2}$, find the relation between x and y , then find y when : $x = 2$ 2018 Exam (21) Question (4) (b)
9	If y = 2 + b where $b \propto x$, $x = 1$ when y = 5 , find the relation between x and y , then find the value of y when $x = 2$ 2018 Exam (8) Question (5) (a)
10	If y = z + 5 , z changes inversely with x and y = 6 when $x = 2$, then find the relation between y and x and find the value of y when $x = 1$ 2018 Exam (6) Question (4) (b)
11	If $4x^2 + 9y^2 = 12xy$, **then prove that :** x varies as y 2018 Exam (25) Question (2) (a)
12	If $\frac{a + 2b}{6} = \frac{b + 3c}{3}$, **prove that :** $a \propto c$ 2018 Exam (15) Question (2) (a)
13	**From the data of the opposite table :** **answer each of the following :** (1) Identify the kind of the variation whether it is direct or inverse. (2) Find the relation between x and y , then find the value of y when $x = 3$ 2018 Exam (1) Question (4) (a)
14	If $a \propto b$ and a = 10 when b = 5 (1) Find the relation between a and b (2) Find the value of b when a = 4 2018 Exam (10) Question (3) (b)

Table for question 13:

x	2	4	6
y	6	3	2

Prep. [3]

First Term

Algebra

Unit [3]

Total

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Prep. [3] - First Term – Algebra – Unit [3] : Statistics

Lesson [1] : Collecting Data

Resources of collecting data is classified into

1 Primary resources (field resources) :

These are the resources from which we get data directly.

2 Secondary resources (historical resources) :

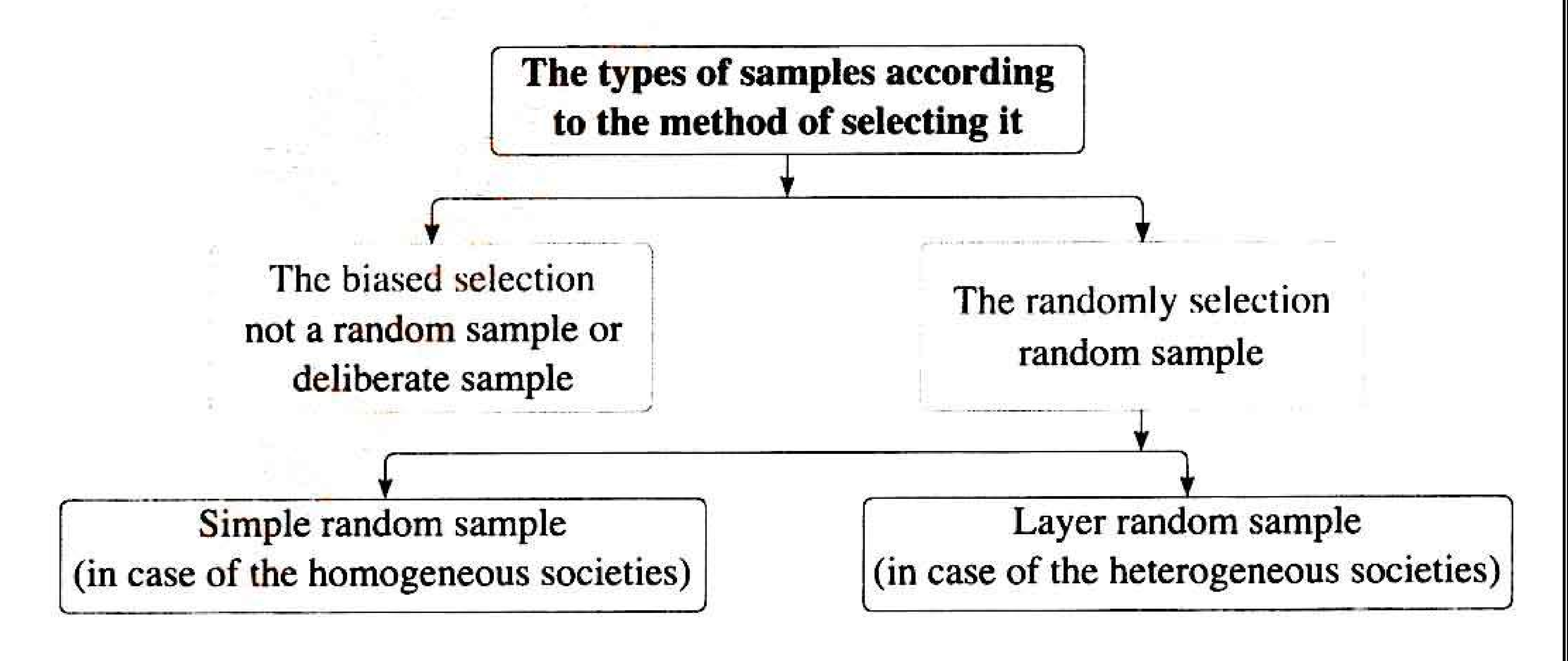
These are the resources from which we get data that previously collected and registered by some authorities , formal organisations or persons.

	1 Primary resources	2 Secondary resources
Examples :	• The personal interview. • Questionnaires (survey). • Observing and measuring.	• Central agency for public mobilization and statistics. • Mass-media and internet. • Documents of data of employees in a company.
Advantages :	Accuracy.	Saves time , effort and money.
Disadvantages :	It needs more time, effort and money besides it requires more investigators in large societies.	It is less accurate.

The concept of the sample

It is a small part from a large society that looks like the society and represents it well.

How can we select the sample ?



Lesson [2] : Dispersion

Prelude

The set A : 29 , 26 , 35 , 35 , 35

The set B : 8 , 35 , 49 , 35 , 33



- The mean = $\frac{\text{the sum of values}}{\text{the number of values}}$
- The median of a set of values is the value which lies at the middle of the set of values after ordering them.
- The mode of a set of values is the most common value in the set.

	mean	median	mode
Set A	32	35	35
Set B	32	35	35

Dispersion of a set of values

It means the divergence or the differences among its values.

- The dispersion is small if the difference among the values is little while the dispersion is great if the difference among the values is great , the dispersion is zero if all the values are equal.

i.e. The dispersion is a measure that expresses how much the sets are homogeneous.

Dispersion measurements

1 The range (the simplest measure of dispersion) :

It is the difference between the greatest value and the smallest value in the set.

The range = the greatest value – the smallest value

For example :

- If the values of set A are 60 , 58 , 62 , 61 and 59 ∴ The range = 62 – 58 = 4
- If the values of set B are 72 , 78 , 46 , 65 and 39 ∴ The range = 78 – 39 = 39

So the set B is more divergent than the set A

The advantages of range :

- It is an easy and simple method that gives a quick idea about the divergence or convergence of the values.
- It is considered as the simplest and the easiest method to measure dispersion.

2 Standard deviation :

It is the most important , common and accurate measure of dispersion. We can calculate it by calculating the positive square root of the average of squares of deviations of the values from their mean. It is denoted by σ and it is read as (sigma).

First 〉 Calculating the standard deviation of a set of values :

$$\text{The standard deviation } \sigma = \sqrt{\frac{\Sigma (x-\overline{x})^2}{n}}$$

Where :

x denotes a value of the values ,

$\overline{x}$ denotes the mean of the values and it is read as x bar ,

n denotes the number of values ,

Σ denotes the summation operation.

Example 1 Calculate the standard deviation of the values : 8 , 9 , 7 , 6 and 5

Solution

1 We find the mean of the values.

$$\overline{x} = \frac{\Sigma x}{n} = \frac{8+9+7+6+5}{5} = 7$$

2 We form the following table :

x	$x-\overline{x}$	$(x-\overline{x})^2$
8	8 – 7 = 1	1
9	9 – 7 = 2	4
7	7 – 7 = 0	0
6	6 – 7 = – 1	1
5	5 – 7 = – 2	4
Total		**10**

3 We calculate the standard deviation by substituting in the law :

$$\text{The standard deviation } (\sigma) = \sqrt{\frac{\Sigma (x-\overline{x})^2}{n}}$$

$$\therefore \text{The standard deviation } (\sigma) = \sqrt{\frac{10}{5}} = \sqrt{2} \simeq 1.4$$

Second 〉 Calculating the standard deviation of a frequency distribution :

For any frequency distribution :

$$\text{The standard deviation } \sigma = \sqrt{\frac{\Sigma (x-\overline{x})^2 k}{\Sigma k}}$$

Where :

x represents the value or the centre of the set ,

k represents the frequence of the value or the set ,

$\sum k$ is the sum of frequences and $\overline{x}$ (the mean) $= \frac{\sum (x \times k)}{\sum k}$

A Calculating the standard deviation of a simple frequency distribution :

Example 2 **The following table shows the distribution of ages of 20 persons in years :**

The age	15	20	22	23	25	30	Total
Number of persons	2	3	5	5	1	4	20

Find the standard deviation of the ages.

Solution

1 We find the mean of the ages $(\overline{x})$ by using the following table :

The age (x)	Number of persons (k)	$x \times k$
15	2	30
20	3	60
22	5	110
23	5	115
25	1	25
30	4	120
Total	**20**	**460**

The mean $(\overline{x}) = \frac{\sum (x \times k)}{\sum k} = \frac{460}{20} = 23$ years.

2 We form the following table :

x	k	$x - \overline{x}$	$(x - \overline{x})^2$	$(x - \overline{x})^2 \times k$
15	2	$15 - 23 = -8$	64	128
20	3	$20 - 23 = -3$	9	27
22	5	$22 - 23 = -1$	1	5
23	5	$23 - 23 = 0$	0	0
25	1	$25 - 23 = 2$	4	4
30	4	$30 - 23 = 7$	49	196
Total	**20**			**360**

3 We calculate the standard deviation from the law :

Standard deviation $(\sigma) = \sqrt{\frac{\sum (x - \overline{x})^2 \times k}{\sum k}} = \sqrt{\frac{360}{20}} = \sqrt{18} \simeq 4.24$ years.

B Calculating the standard deviation of a frequency distribution of sets :

Example 3 **The following is the frequency distribution of weekly incentives of 100 workers in a factory :**

Incentives in pounds	35 –	45 –	55 –	65 –	75 –	85 –
Number of workers	10	14	20	28	20	8

Find the standard deviation of this distribution.

Solution

1 We find the mean $(\overline{x})$ by using the following table :

Remember that :
The centre of the set $= \frac{\text{lower limit + upper limit}}{2}$

Sets	Centres of sets (x)	Frequence (k)	$x \times k$
35 –	40	10	400
45 –	50	14	700
55 –	60	20	1200
65 –	70	28	1960
75 –	80	20	1600
85 –	90	8	720
Total		**100**	**6580**

$\therefore$ The mean $(\overline{x}) = \frac{\Sigma (x \times k)}{\Sigma k} = \frac{6580}{100} = 65.8$ pounds.

2 We form the following table :

x	k	$x - \overline{x}$	$(x - \overline{x})^2$	$(x - \overline{x})^2 \times k$
40	10	$40 - 65.8 = -25.8$	665.64	6656.4
50	14	$50 - 65.8 = -15.8$	249.64	3494.96
60	20	$60 - 65.8 = -5.8$	33.64	672.8
70	28	$70 - 65.8 = 4.2$	17.64	493.92
80	20	$80 - 65.8 = 14.2$	201.64	4032.8
90	8	$90 - 65.8 = 24.2$	585.64	4685.12
Total	**100**			**20036**

3 We calculate the standard deviation by using the law :

Standard deviation $(\sigma) = \sqrt{\frac{\Sigma (x - \overline{x})^2 \times k}{\Sigma k}} = \sqrt{\frac{20036}{100}} \simeq 14.15$ pounds.

Remarks

- From the previous , we notice that the standard deviation is influenced by all values not by the two terminal values only (the smallest and the greatest value) as the range , therefore it represents the dispersion well.
- The standard deviation has the same measuring units of the original data.
- The values which are more homogeneous have less dispersion and their standard deviation is small.
- If the standard deviation equals zero that means the all values are equal , it is the perfect homogeneous case (the vanished dispersion).

Exercises

[A] : Choose The Correct Answer : -

1	The simplest and easiest method of measuring dispersion is the .... (a) arithmetic mean. (b) mode. (c) median. (d) range.
2	Which of the following is a measure for dispersion ? (a) The median. (b) The mean. (c) The range. (d) The mode.
3	The commonest measure of dispersion and the most accurate is the .... (a) range. (b) mode. (c) standard deviation. (d) median.
4	.... is one of the measures of the dispersion. (a) The median (b) The arithmetic mean (c) The standard deviation (d) The mode
5	The difference between the greatest value and the smallest value of a set of data is .... (a) the mode (b) the median (c) the arithmetic mean (d) the range
6	The positive square root of mean of the squares of deviations of values from its arithmetic mean is called .... (a) the range. (b) the arithmetic mean. (c) the standard deviation. (d) the mode.

7	Selection a sample of layers of a statistical society is called a .... sample. (a) random (b) bunch (c) deliberate (d) class (layer)
8	The range of the set of the values 3 , 7 , 9 and 10 is .... (a) 4 (b) 5 (c) 7 (d) 6
9	The range of the set of the values 3 , 17 , 12 , 30 and 28 is .... (a) 3 (b) 27 (c) 33 (d) 30
10	The range of the set of the values 5 , 5 , 5 , 5 , 5 equals .... (a) zero (b) 5 (c) 15 (d) 20
11	The range of the set of values : 5 , 14 , 4 , 23 , 15 is .... (a) 12 (b) 14 (c) 19 (d) 23
12	The range of the set of values : 5 , 14 , 4 , 21 , 16 , 12 is .... (a) 21 (b) 16 (c) 17 (d) 15
13	The range of the set of the values : 7 , 3 , 6 , 9 and 5 is .... (a) 3 (b) 4 (c) 6 (d) 12
14	The range of the set of the values 7 , 5 , 2 , 9 , 15 and 17 is .... (a) 5 (b) 10 (c) 15 (d) 20
15	The range of the set of the values 7 , 5 , 3 , 10 and 15 is .... (a) 3 (b) 8 (c) 12 (d) 15
16	The range of the set of the values 7 , 16 , 14 , 9 and 5 equals .... (a) 5 (b) 14 (c) 11 (d) 21
17	The range of the set of values 8 , 5 , 10 , 6 , 14 is .... (a) 3 (b) 9 (c) 6 (d) 14
18	The range of the values 8 , 12 , 20 , 17 , 13 is .... (a) 8 (b) 12 (c) 13 (d) 17
19	The range of the set of values 20 , 20 , 25 , 35 and 55 is .... (a) 20 (b) 25 (c) 31 (d) 35
20	If the range of the values 2 , 7 , 6 , x is 8 , where $x > 0$, then $x =$ .... (a) 4 (b) 9 (c) 10 (d) – 1
21	The arithmetic mean of the set of the values 5 , 9 , 2 , 10 and 4 equals .... (a) 5 (b) 6 (c) 10 (d) 4

22	The arithmetic mean of the set of the values : 7 , 3 , 6 , 9 and 5 equals .... (a) 3 (b) 6 (c) 4 (d) 12
23	If the arithmetic mean for the set of the values 7, x , 9 , 11 is 8 , then x = ... (a) 8 (b) 7 (c) 6 (d) 5
24	If the mean of the values 3 , 5 , 7 , x is 6 , then x = ... (a) 9 (b) 3 (c) 8 (d) 6
25	If the mean for the values x , 2 x , 3 x and 4 x is 5 , then x = ... (a) 1 (b) 2 (c) 3 (d) 4
26	If the arithmetic mean of the set of the values 5 , 6 , 7 , a , 8 equals 6 , then a = ... (a) 8 (b) 4 (c) 26 (d) 30
27	The arithmetic mean of the set of the values 2 k , 3 , 5 where k $\in \mathbb{R}^+$ is 10 , then k = ... (a) 15 (b) 13 (c) 12 (d) 11
28	If the arithmetic mean of a set of the values : a , 5 , 8 , 7 , 6 equals 6 , then a = ... (a) 3 (b) 4 (c) 5 (d) 6
29	If $2x + 2y = 10$, $x , y \in \mathbb{R}^+$, then the arithmetic mean of the values x , y equals .... (a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) 5 (d) 2
30	The standard deviation of the values 3 , 3 , 3 and 3 is .... (a) zero (b) 3 (c) 4 (d) 12
31	If all the individuals are equal in values , then .... (a) $\overline{x} = 0$ (b) $\sigma = 0$ (c) $x - \overline{x} > 0$ (d) $x - \overline{x} < 0$
32	If $\sum (x - \overline{x})^2 = 36$ for a set of values whose number is 9 , then σ = ... (a) 2 (b) 4 (c) 18 (d) 27
33	If $\sum (x - \overline{x})^2 = 144$ for a set of values whose number is 9 , then σ = ... (a) 9 (b) 16 (c) 4 (d) 135
34	If $\sum (x - \overline{x})^2 = 48$ for a set of values whose number is 12 , then σ = ... (a) – 4 (b) – 2 (c) 2 (d) 4
35	If 67 is the greatest value of a set and if the range equals 27 , then the smallest value of this set equals .... (a) 67 (b) 40 (c) 27 (d) 94

36 If 18 is the greatest individual of a set of individuals and its range is 6 , then the smallest individual of this set = ...

(a) 8 (b) 12 (c) 24 (d) 36

37 If 9 is the greatest individual of a set of individuals and its range is 6 , then the smallest individual of this set equals = ...

(a) 3 (b) 6 (c) 9 (d) 15

38 If 65 is the greatest individuals of a set of individuals and its range is 29 , then the smallest individual of this set equals ....

(a) 35 (b) 37 (c) 38 (d) 36

39 The set which has the greatest dispersion of the following sets is ....

(a) 28 , 17 , 30 , 36 , 20 (b) 25 , 39 , 19 , 5 , 27

(c) 20 , 19 , 29 , 37 , 43 (d) 31 , 35 , 26 , 37 , 41

40 Which of the following values of a to make the range of the numbers 53 , a , 58 , 57 , 60 and 55 is equal to 9 ?

(a) 63 (b) 61 (c) 51 (d) 50

41 A factory has 125 workers , 75 technicians and 50 engineers. It is wanted to take a sample of layers of size 50 individuals such that it represents each layer due to its size , then the number of engineers of the sample = ...

(a) 30 (b) 20 (c) 25 (d) 15

[C] : Essay Problems : -

1 **Calculate the arithmetic mean and the standard deviation of the following data :**

3 , 12 , 17 , 28 , 30

2017 Exam (3) Question (5) (b)

2 Calculate the arithmetic mean of the set of values :

3 , 5 , 7 , 9 and 11 , then find the standard deviation of this values.

2018 Exam (1) Question (3) (b)

3 The following values represent marks of five pupils in a test : 8 , 9 , 6 , 12 , 10

Calculate : ① The mean of the marks.

② The standard deviation of the marks.

2018 Exam (8) Question (3) (b)

4

The following is the frequency distribution for a number of defective units which found in 100 boxes of manufactured units :

Number of defective units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation of the defective units.

2018 Exam (22) Question (5) (b)

5

The following table shows the frequency distribution of the marks of 20 students in an exam :

Set	0 –	2 –	4 –	6 –	8 – 10
Frequency	1	3	6	5	5

Calculate the standard deviation.

2017 Exam (9) Question (5) (b)

6

Calculate each of the arithmetic mean and the standard deviation of the following :

The set	0 –	2 –	4 –	6 –	8 –
The frequency	5	9	15	15	6

2017 Exam (13) Question (5) (b)

7

Calculate each of the arithmetic mean and the standard deviation of the following data :

The Set	0 –	10 –	20 –	30 –	40 –	Total
Frequency	2	3	18	7	10	40

2017 Exam (10) Question (5) (b)

8

Calculate the arithmetic mean and the standard deviation of the following frequency distribution :

Sets	Zero –	4 –	8 –	12 –	16 – 20	Total
Frequency	3	4	7	2	9	25

2018 Exam (7) Question (5) (b)

9

The following frequency distribution shows the ages of 10 children :

Ages in year	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation of ages in years.

2018 Exam (3) Question (5) (b)

Prep. [3]

First Term

Geometry

Unit [4]

Lesson [1]

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Prep. [3] - First Term – Geometry – Unit [4] : Trigonometry

Lesson [1] : The Man Trigonometrical Ratios Of The Acute Angle

Prelude

The relation between each of the degrees, the minutes and the seconds

- **The degree = 60 minutes.**
- **The minute = 60 seconds**

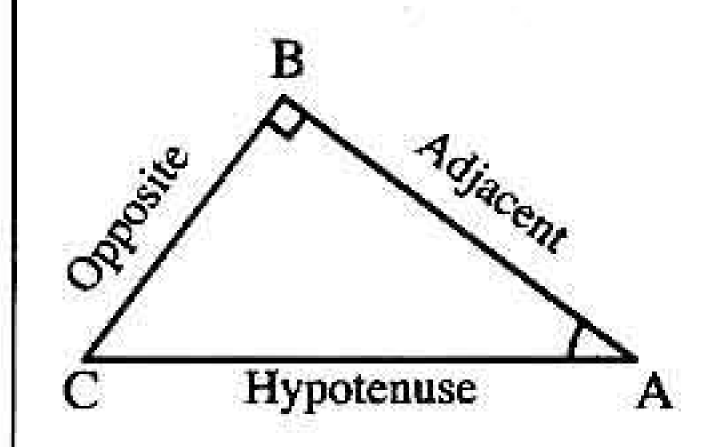
i.e. **The degree = 60 × 60 = 3600 seconds.**

The main trigonometrical ratios of the acute angle

The trigonometrical ratio of the acute angle

It is the ratio between two side lengths of the right-angled triangle that contains this angle.

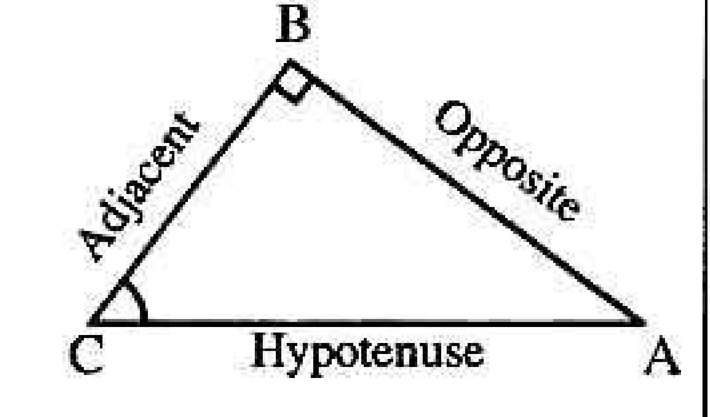
i.e. **If Δ ABC is a right-angled triangle at B , then :**



According to angle A

1. $\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$
2. $\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC}$
3. $\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB}$

According to angle C



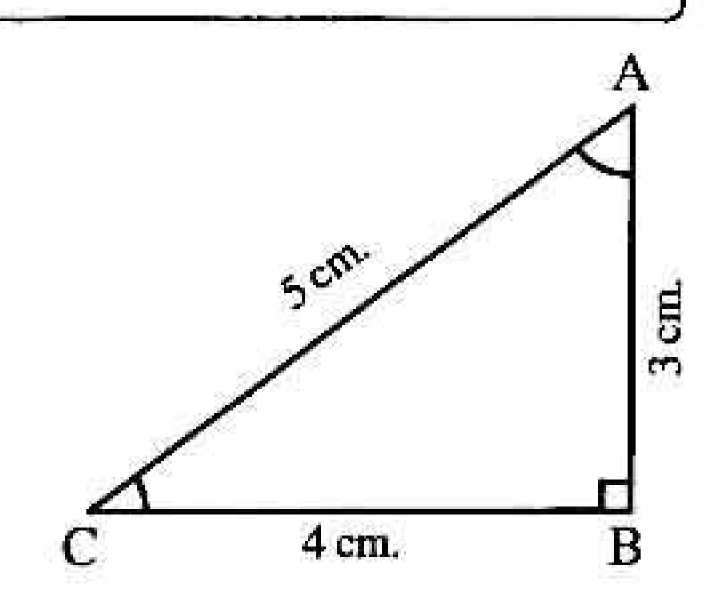
1. $\sin C = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC}$
2. $\cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BC}{AC}$
3. $\tan C = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AB}{BC}$

For example :

In the opposite figure :

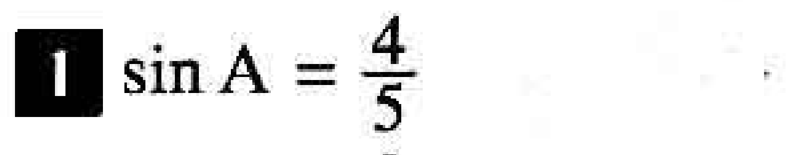
If Δ ABC is a right-angled triangle at B ,

AB = 3 cm. , BC = 4 cm. and AC = 5 cm. , then :



1. $\sin A = \frac{4}{5}$
2. $\cos A = \frac{3}{5}$
3. $\tan A = \frac{4}{3}$

1. $\sin C = \frac{3}{5}$
2. $\cos C = \frac{4}{5}$
3. $\tan C = \frac{3}{4}$



In the previous example, note that :

1. $\sin B = \cos C = \frac{4}{5}$, $\sin C = \cos B = \frac{3}{5}$

   and by noticing : m (∠ B) + m (∠ C) = 90° "Complementary angles"

We can deduce that :

The sine of any acute angle equals the cosine of its complementary

i.e. If m (∠ A) + m (∠ B) = 90° , then sin A = cos B and sin B = cos A

and vice versa ***i.e.*** If ∠ A and ∠ B are acute angles and sin A = cos B

then m (∠ A) + m (∠ B) = 90°

2 $\frac{\sin B}{\cos B} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$, $\tan B = \frac{4}{3}$ $\therefore \tan B = \frac{\sin B}{\cos B}$

, $\frac{\sin C}{\cos C} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$, $\tan C = \frac{3}{4}$ $\therefore \tan C = \frac{\sin C}{\cos C}$

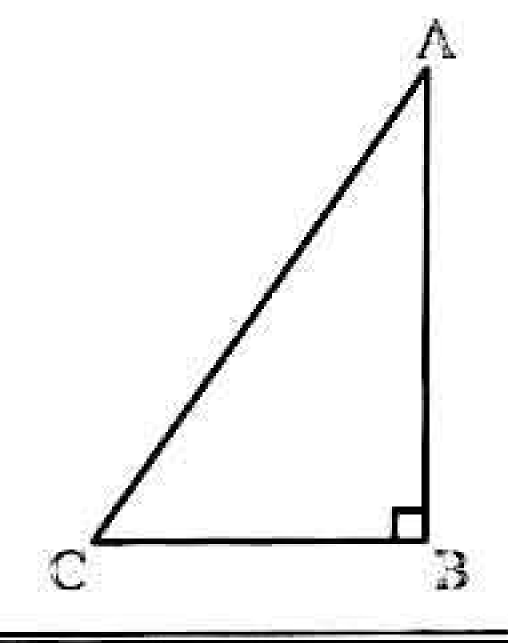
Generally

$$\text{The tangent of the angle} = \frac{\text{The sine of the angle}}{\text{The cosine of the angle}}$$

Remember Pythagoras' theorem :

If ABC is a right-angled triangle at B , then :

- $(AC)^2 = (AB)^2 + (BC)^2$
- $(AB)^2 = (AC)^2 - (BC)^2$
- $(BC)^2 = (AC)^2 - (AB)^2$



Exercises

[A] : Choose The Correct Answer : -

1	If XYZ is a right-angled triangle at Y , then sin Z = …………… (a) $\frac{YZ}{XZ}$ (b) $\frac{XY}{YZ}$ (c) $\frac{XY}{XZ}$ (d) $\frac{YZ}{XY}$
2	In Δ ABC , m (∠ B) = 90° , then sin A – cos C = …………… (a) 2 sin A (b) 2 cos C (c) 2 cos A (d) 0
3	If Δ ABC is a right-angled triangle at B , then sin C + cos C …………… 1 (a) = (b) > (c) < (d) ≤
4	ABC is a right-angled triangle at B , then sin A + cos C = …………… (a) 2 sin C (b) 2 sin B (c) 2 cos A (d) 2 sin A

5 If $\sin\theta = \cos\theta$, then m ($\angle\theta$) = ...°

(a) 30 (b) 45 (c) 60 (d) 75

6 In Δ ABC , if sin A = cos B , then Δ ABC is ....

(a) obtuse-angled. (b) acute-angled.

(c) right-angled. (d) obtuse-angled and isosceles.

7 If $\sin X = 0.8$ where X is the measure of an acute angle , then $X \simeq$ ....°

(a) 53 (b) 37 (c) 39 (d) 83

8 If X , y are measures of two complementary angles and $\sin X = \frac{3}{5}$, then cos y = ...

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

9 **In the opposite figure :**

$\overline{AB}$ is a diameter of a circle , then the surface area of the shaded shape = ... cm^2

(a) 4π (b) 16π

(c) 2π (d) 9π

10 **In the opposite figure :** sin A + sin C = ...

(a) 2 (b) $\frac{31}{25}$

(c) $\frac{17}{25}$ (d) 1

11 **In the opposite figure :** Δ ABC is a right-angled at B ,

$\overline{BE}$ is a median , BE = 5 cm. , AB = 6 cm. , then sin C = ...

(a) $\frac{5}{6}$ (b) $\frac{3}{5}$

(c) $\frac{6}{5}$ (d) $\frac{5}{3}$

12 If m ($\angle$ A) = 85° , sin B = cos B in Δ ABC , then m ($\angle$ C) = ...

(a) 30° (b) 45° (c) 50° (d) 60°

13 In Δ ABC , m ($\angle$ C) = 90° , if AB = 15 cm. , BC = 9 cm. , then AC = ... cm.

(a) 6 (b) 24 (c) 12 (d) 36

14 In the triangle ABC , if m ($\angle$ A) : m ($\angle$ B) : m ($\angle$ C) = 3 : 4 : 5 , then cos B = ...

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

15 If the point ($X - 5$, $3 - X$) lies in the third quadrant , then X = ...

(a) 7 (b) 6 (c) 5 (d) 4

16	The tangent of an acute angle of the right isosceles triangle is equal to …………… (a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\frac{\sqrt{2}}{2}$
17	DEF is a right-angled triangle at E , which of the following relations is false ? (a) tan D × tan F = 1 (b) sin D = cos F (c) cos D = sin F (d) cos D = sin E
18	If x and y are the measures of two complementary angles where x : y = 2 : 1 , then sin x + cos y = …………… (a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{3}$

[B] : Essay Problems : -

1	Two angles A and B are complementary , if the ratio between their measures is 2 : 1 **Find :** sin A + cos B 2018 Exam (17) Question (5) (b)
2	ABC is a right-angled triangle at B , AB = 3 cm. , BC = 4 cm. (1) **Find the value of each of :** tan C , sin A (2) **Prove that :** $\sin^2 C + \cos^2 C = 1$ 2017 Exam (5) Question (2) (a)
3	ABC is a right-angled triangle at C , AB = 10 cm. , BC = 8 cm. , **Find the value of :** sin A cos B + cos A sin B 2017 Exam (11) Question (2) (a)
4	ABC is a triangle in which : AB = AC = 10 cm. , BC = 12 cm. Draw : $\overrightarrow{AD} \perp \overline{BC}$, $\overrightarrow{AD} \cap \overline{BC} = \{D\}$ **Prove that :** $\sin^2 C + \cos^2 C = 1$ 2018 Exam (3) Question (5) (a)
5	ABC is a right-angled triangle at B where AC = 7.52 cm. and m (∠ C) = 53° **Find :** the perimeter of Δ ABC to the nearest cm. 2018 Exam (9) Question (2) (b)
6	ABC is an isosceles triangle in which AB = AC = 10 cm. , BC = 12 cm. and $\overline{AD} \perp \overline{BC}$ **Find :** (1) The measure of ∠ B (2) The surface area of Δ ABC 2018 Exam (22) Question (3) (b)
7	2017 Exam (20) Question (3) (b)

ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$, m (∠ B) = 90°

If AB = 6 cm. , AD = 12 cm. , BC = 20 cm.

, find the value of : cos (∠ DCB) – tan (∠ ACB)

8 **In the opposite figure :**

Δ ABC in which m (∠ B) = 90°

, AB = 3 cm. and BC = 4 cm.

Prove that : sin A cos C + cos A sin C = 1

2018 Exam (17) Question (3) (b)

9 **In the opposite figure :**

m (∠ B) = 90° , AB = 5 cm.

and AC = 13 cm.

Find : cos A cos C – sin A sin C

2018 Exam (19) Question (5) (a)

10 **In the opposite figure :**

ABC is a right-angled triangle at B , AB = 6 cm.

and BC = 8 cm. **, then find :**

(1) The length of $\overline{AC}$

(2) sin A + cos A

2018 Exam (23) Question (4) (a)

11 **In the opposite figure :**

CAD is a right-angled triangle at A

, CD = 35 cm. , sin D = $\frac{3}{5}$

Calculate the length of $\overline{AC}$ and the perimeter of Δ CAD

2017 Exam (5) Question (4) (b)

12 **In the opposite figure :**

ABCD is a rectangle where : BC = 8 cm. and AC = 10 cm.

Find : (1) m (∠ ACB)

(2) The surface area of the rectangle ABCD

2018 Exam (10) Question (4) (a)

13	**In the opposite figure :** ABCD is a rectangle where AB = 4 cm. , BC = 8 cm. and H is the midpoint of $\overline{BC}$ **Find the value of :** tan (∠ AHB) + tan (∠ ACD) 2018 Exam (9) Question (4) (b)
14	**In the opposite figure :** $\overline{AD} \perp \overline{BC}$, AB = 15 cm. , BD = 9 cm. and DC = 5 cm. **Find :** (1) m (∠ B) (2) The value of sin (∠ BAD) + tan (∠ C) 2017 Exam (9) Question (2) (b)
15	**In the opposite figure :** m (∠ A) = 90° , $\overline{DH} \perp \overline{BC}$ where H is the midpoint of $\overline{BC}$, AD = 5 cm. and BD = 13 cm. **Find with proof :** tan B 2018 Exam (12) Question (5) (b)
16	ABC is a right-angled triangle at C , AB = 13 cm. , BC = 12 cm. (1) **Prove that :** sin A cos B + cos A sin B = 1 (2) **Find :** $1 + \tan^2 A$ 2017 Exam (13) Question (1) (b)

Prep. [3]

First Term

Geometry

Unit [4]

Lesson [2]

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Prep. [3] - First Term – Geometry – Unit [4] : Trigonometry

Lesson [2] : The Man Trigonometrical Ratios Of Some Angles

The following table summarizes the trigonometrical ratios of the angles measuring 30° , 60° and 45° :

The measure of the angle / The trigonometrical ratio	30°	60°	45°
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

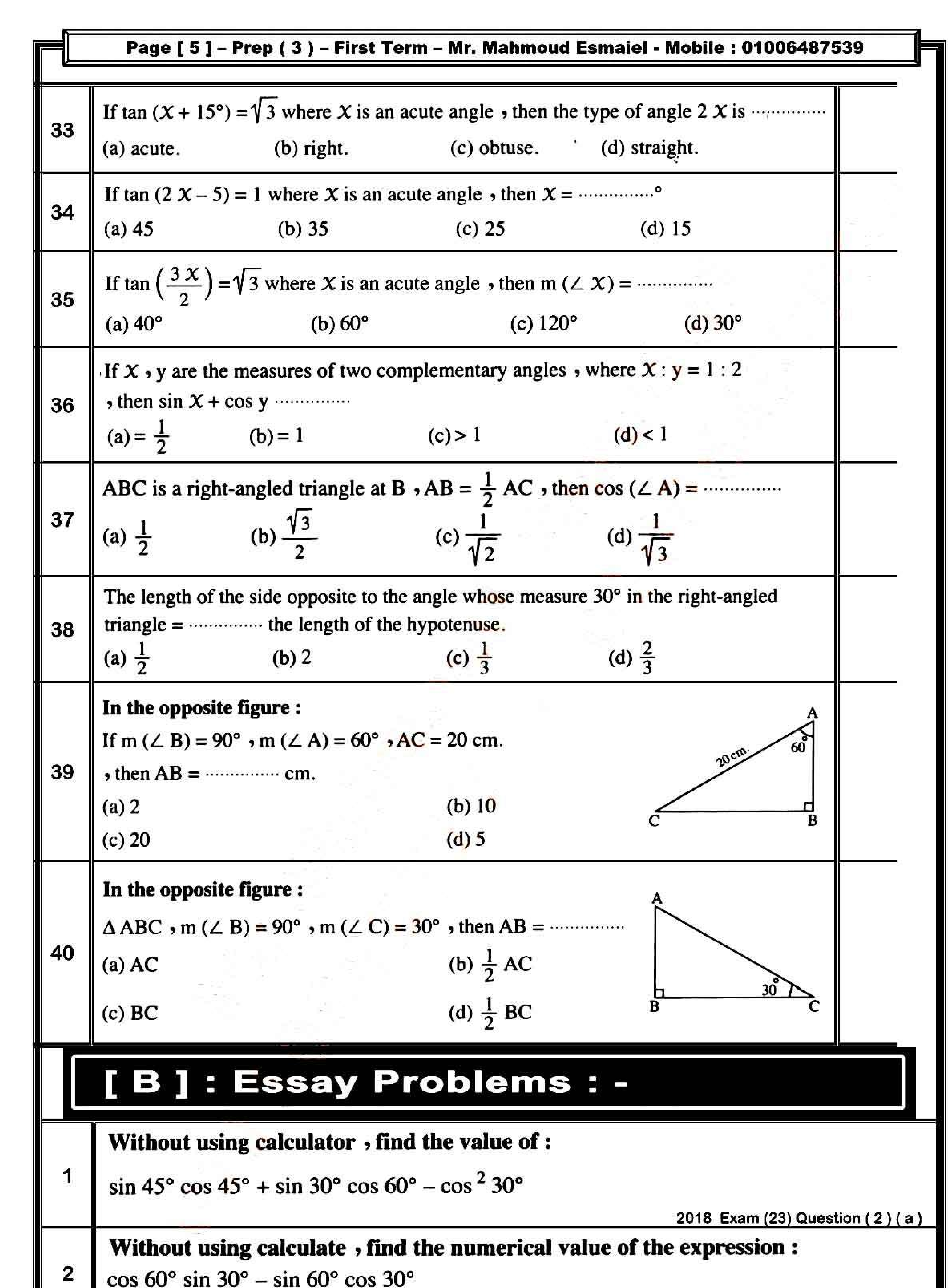
Exercises

[A] : Choose The Correct Answer : -

1	$\tan 45° =$ .... (a) 1 (b) $2\sqrt{2}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$
2	$\cos 60° =$ .... (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$
3	$\cos 60° + \sin 30° =$ .... (a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{2}$ (d) 1
4	$\tan 45° + \sin 30° =$ .... (a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$
5	$\sin 30° - \cos 60° =$ .... (a) 1 (b) $\frac{1}{2}$ (c) zero (d) $\sqrt{3}$
6	$\cos^2 30° - \sin^2 30° =$ .... (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

7	tan 45° sin 30° = …………… (a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{1}{4}$
8	sin 45° cos 45° = …………… (a) 2 (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$
9	2 cos 60° = …………… (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$
10	2 sin 30° tan 60° = …………… (a) $\sqrt{3}$ (b) 3 (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{1}{2}$
11	2 sin 30° cos 30° = …………… (a) $\frac{1}{2}$ (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$
12	2 sin 60° cos 60° = …………… (a) sin 60° (b) cos 60° (c) tan 60° (d) sin 30°
13	4 cos 30° tan 60° = …………… (a) 3 (b) $2\sqrt{3}$ (c) 6 (d) 12
14	If sin X = $\frac{1}{2}$, X is an acute angles , then m (∠ X) = …………… (a) 45° (b) 60° (c) 30° (d) 90°
15	If sin X = 0.5 where X is the measure of an acute angle , then X = ……………° (a) 150 (b) 60 (c) 45 (d) 30
16	If sin X = $\frac{1}{2}$ where X is an acute angle , then sin 2 X = …………… (a) $\frac{1}{4}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$
17	If sin 30° = cos θ where θ is an acute angle , then m (∠ θ) = ……………° (a) 15 (b) 30 (c) 60 (d) 90
18	If cos X = $\frac{\sqrt{3}}{2}$, X is an acute angle , then sin 2 X = …………… (a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) – 2 (d) $\frac{1}{\sqrt{3}}$
19	If tan X = $\sqrt{3}$, where ∠ X is acute , then m (∠ X) = ……………° (a) 60 (b) 10 (c) 20 (d) 30

20	If $\cos 2X = \frac{1}{2}$ where X is an acute angle , then m ($\angle X$) = ... (a) 15° (b) 30° (c) 45° (d) 60°
21	If $\sin 2X = 0.5$ where X is an acute angle , then m ($\angle X$) = ...° (a) 70 (b) 60 (c) 15 (d) 30
22	If $\cos(3X) = \frac{1}{2}$, where ($3X$) is an acute angle , then X = ...° (a) 15 (b) 20 (c) 30 (d) 45
23	If $\tan \frac{3X}{2} = 1$ where X is a measure of an acute angle , then X = ...° (a) 15 (b) 30 (c) 45 (d) 60
24	$2 \sin 30° \cos 30° = \sin$ .... (a) 30° (b) 45° (c) 60° (d) 15°
25	Δ ABC is right-angled at A , if tan B = 1 , then tan C – sin C cos C = ... (a) zero (b) 1 (c) 2 (d) $\frac{1}{2}$
26	If $\sin \frac{X}{2} = \frac{1}{2}$ where X is an acute angle , then m ($\angle X$) = ... (a) 30° (b) 60° (c) 15° (d) 45°
27	If $\cos \frac{X}{2} = \frac{1}{2}$ where $\frac{X}{2}$ is an acute angle , then m ($\angle X$) = ... (a) 100° (b) 120° (c) 130° (d) 110°
28	If $\cos \frac{X}{2} = \frac{\sqrt{3}}{2}$ where X is an acute angle , then sin X = ... (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$
29	If $\tan 3X = \sqrt{3}$ where X is an acute angle , then m ($\angle X$) = ... (a) 20° (b) 50° (c) 60° (d) 30°
30	If $\tan 3X = 1$ where $3X$ is the measure of an acute angle , then m ($\angle X$) = ...° (a) 5 (b) 10 (c) 15 (d) 45
31	If $\sin(X + 10°) = \frac{1}{2}$ where X is an acute angle , then m ($\angle X$) = ...° (a) 10 (b) 20 (c) 30 (d) 40
32	If $\tan(X + 10)° = 1$ where X is an acute angle , then m ($\angle X$) = ... (a) 35° (b) 45° (c) 11° (d) 40°



33 If $\tan(X + 15°) = \sqrt{3}$ where X is an acute angle , then the type of angle $2X$ is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

34 If $\tan(2X - 5) = 1$ where X is an acute angle , then $X =$ ....°

(a) 45 (b) 35 (c) 25 (d) 15

35 If $\tan\left(\frac{3X}{2}\right) = \sqrt{3}$ where X is an acute angle , then $m(\angle X) =$ ....

(a) 40° (b) 60° (c) 120° (d) 30°

36 If X , y are the measures of two complementary angles , where $X : y = 1 : 2$, then $\sin X + \cos y$ ....

(a) $= \frac{1}{2}$ (b) $= 1$ (c) > 1 (d) < 1

37 ABC is a right-angled triangle at B , $AB = \frac{1}{2} AC$, then $\cos(\angle A) =$ ....

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

38 The length of the side opposite to the angle whose measure 30° in the right-angled triangle = ... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

39 **In the opposite figure :**

If $m(\angle B) = 90°$, $m(\angle A) = 60°$, AC = 20 cm.

, then AB = ... cm.

(a) 2 (b) 10

(c) 20 (d) 5

40 **In the opposite figure :**

ΔABC , $m(\angle B) = 90°$, $m(\angle C) = 30°$, then AB = ...

(a) AC (b) $\frac{1}{2}$ AC

(c) BC (d) $\frac{1}{2}$ BC

[B] : Essay Problems : -

1 **Without using calculator , find the value of :**

$\sin 45° \cos 45° + \sin 30° \cos 60° - \cos^2 30°$

2018 Exam (23) Question (2) (a)

2 **Without using calculate , find the numerical value of the expression :**

$\cos 60° \sin 30° - \sin 60° \cos 30°$

2018 Exam (19) Question (3) (b)

3	**Without using the calculator , find the numerical value of the following expression :** $\tan^2 60° - 2 \sin 45° \cos 45°$ 2017 Exam (1) Question (2) (a)
4	**Without using the calculator , find :** $\frac{\sin 30°}{\cos 60°} - \cos 30° \sin 60°$ 2018 Exam (9) Question (3) (b)
5	**Without using the calculator , prove that :** $3 \sin 30° = 5 \cos 60° - \tan^2 45°$ 2017 Exam (9) Question (2) (a)
6	**Without using calculator , prove that :** $\cos 60° = \cos^2 30° - \sin^2 30°$ 2018 Exam (3) Question (2) (a)
7	**Prove that :** $\tan 60° (1 - \tan^2 30°) = 2 \tan 30°$ 2018 Exam (15) Question (4) (a)
8	**Without using the calculator , prove that :** $\sin^2 60° = 2 \sin 30° \cos^2 30°$ 2018 Exam (5) Question (2) (a)
9	**Prove that :** $\tan 45° = \sin^2 30° + \sin^2 60°$ 2018 Exam (24) Question (2) (a)
10	**Prove that :** $\sin^3 30° = 9 \cos^3 60° - \tan^2 45°$ 2017 Exam (16) Question (2) (a)
11	If $\sin^2 45° = \cos \theta \tan 30°$ **Find :** m (∠ θ) where θ is an acute angle. 2018 Exam (6) Question (2) (a)
12	2017 Exam (18) Question (2) (a) (1) **Find the value of** X **if :** $X \cos 30° = \tan 60°$ (2) **Find m (∠ θ) , where θ is an acute angle if :** $\sin^2 45° = \cos \theta \tan 30°$
13	**Find the value of** X **in degrees , where** $0° < X < 90°$ **if :** $\sin X = \sin 60° \cos 30° - \cos 60° \sin 30°$ 2018 Exam (1) Question (5) (a)
14	If $\sin X = 3 \sin 30° \cos 60°$, then find the value of X to the nearest minutes such that X is an acute angle. 2018 Exam (8) Question (3) (a)
15	Without using the calculator , find : $\cos X$, if $2 \sin X = \tan^2 60° - 2 \tan 45°$ where X is the measure of an acute angle. 2018 Exam (5) Question (3) (a)
16	Find to the nearest minute the numerical value of y where $\cos y = \frac{4}{3} - 2 \sin^2 45°$, such that y is a measure of an acute angle. 2017 Exam (8) Question (4) (b)

17	If $4 \cos 60° \sin 30° = \tan X$, find the value of X , where X is an acute angle. 2017 Exam (1) Question (3) (a)
18	If $\tan X = 4 \sin 30° \cos 30°$ **Find :** m ($\angle X$) where X is an acute angle , showing steps of the solution. 2018 Exam (7) Question (2) (b)
19	**Find m ($\angle \theta$) where θ is an acute angle :** $3 \tan^2 \theta = 4 \sin^2 30° + 8 \cos^2 60°$ 2018 Exam (13) Question (4) (a)
20	**Without using calculator , find the value of :** $\sin^2 60° - \tan 60° \cos 30° + \cos 60° \sin 30°$ 2018 Exam (18) Question (3) (a)
21	**Without using calculator , find the numerical value of the expression :** $(\cos 30° - \cos 60°)(\sin 60° + \sin 30°)$ 2018 Exam (20) Question (3) (a)
22	**Without using calculator , find the value of :** $\frac{\sin 30° \cos 45° + \cos 30° \sin 45°}{\sin 45° \cos 60° + \sin 45° \sin 60°}$ 2017 Exam (10) Question (2) (a)

Prep. [3]

First Term

Geometry

Unit [5]

Lesson [1]

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Unit [5] : Analytical Geometry

Lesson [1] : Distance Between Two Points

Let M (x_1 , y_1) and N (x_2 , y_2) be two points in the same coordinates plane.

i.e. The distance between the two points M and N equals $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- The distance between the two points M (3 , 6) and N (– 1 , 4) is :

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1-3)^2 + (4-6)^2} = \sqrt{(-4)^2 + (-2)^2}$$

$$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \text{ length unit.}$$

Remark [1]

To prove that three given points are collinear (***i.e.*** they lie on one straight line) we can find the distance between each two of these points , then prove that the greatest distance equals the sum of the two other distances.

Remark [2]

- To prove that the points : A , B and C are vertices of a triangle , we can find AB , BC and AC , then prove that the sum of the smaller two lengths is greater than the third length.
- To determine the type of the triangle ABC according to its angle measures (where $\overline{AC}$ is the longest side of the triangle ABC)

  We compare between $(AC)^2$ and $(AB)^2 + (BC)^2$ as the following :

  1 If $(AC)^2 > (AB)^2 + (BC)^2$, then the triangle is obtuse-angled at B

  2 If $(AC)^2 = (AB)^2 + (BC)^2$, then the triangle is right-angled at B

  3 If $(AC)^2 < (AB)^2 + (BC)^2$, then the triangle is acute-angled.

Remark [3]

If ABCD is a quadrilateral :

1 To prove that ABCD is a parallelogram , we prove that : AB = CD , BC = AD

2 To prove that ABCD is a rhombus , we prove that : AB = BC = CD = DA

3 To prove that ABCD is a rectangle , we prove that : AB = CD , BC = AD , AC = BD

4 To prove that ABCD is a square , we prove that : AB = BC = CD = DA , AC = BD

Remark [4]

- To prove that : Three points as A , B and C lie on a same circle of centre M

  we prove that : MA = MB = MC

Exercises

[A] : Choose The Correct Answer : -

1	The distance between the two points (0 , 0) , (2 , 3) equals .... length unit. (a) $\sqrt{5}$ (b) $\sqrt{13}$ (c) $\sqrt{7}$ (d) $\sqrt{11}$
2	The distance between the two points (0 , 0) , (3 , – 4) equals .... length units. (a) 1 (b) 5 (c) – 1 (d) 7
3	The distance between the point (– 3 , 4) and the origin point on a perpendicular coordinate plane = ... length unit. (a) – 5 (b) 25 (c) $\frac{1}{5}$ (d) 5
4	If O (0 , 0) and A (3 , 4) , then the length of $\overline{OA}$ = ... length unit. (a) 3 (b) 4 (c) 5 (d) 7
5	The length of the line segment which is drawn between the two points (0 , 0) , (– 4 , 3) equals .... length unit. (a) 3 (b) 4 (c) $\sqrt{7}$ (d) 5
6	The distance between the point (4 , – 3) and the origin point equals .... length units. (a) – 3 (b) 3 (c) 4 (d) 5
7	The distance between the point (4 , 3) and the origin point = ... length unit. (a) 1 (b) 3 (c) 4 (d) 5
8	The length of the line segment which is drawn between the two points (0 , 0) , (5 , 12) equals .... length unit. (a) 5 (b) 7 (c) 12 (d) 13
9	The distance between the point (1 , 5) and the X-axis is .... length unit. (a) 1 (b) 4 (c) 5 (d) 6
10	The distance between the point (4 , 3) and X-axis is .... (a) – 3 (b) 3 (c) 4 (d) – 4
11	The point (2 , – 4) lies at distance from X-axis = ... length unit. (a) 4 (b) 2 (c) – 4 (d) 6
12	The distance between the point (3 , – 4) and the X-axis is .... length unit. (a) – 3 (b) 4 (c) – 4 (d) 3

13	The distance between the point (4 , – 3) and the X-axis = ... length unit. (a) – 3 (b) 3 (c) 4 (d) 5
14	The distance between the point (5 , $\tan^2 60°$) and the X-axis = ... length unit. (a) 3 (b) $\sqrt{5}$ (c) 3 (d) $\sqrt{3}$
15	The distance between the point (5 , – 2) and the X-axis equals .... length unit. (a) – 2 (b) 2 (c) 3 (d) 7
16	The distance between the point (3 , 4) and the y-axis is .... length unit. (a) 5 (b) 3 (c) 4 (d) 7
17	The distance between the point (– 3 , 4) and y-axis equals .... length units. (a) 1 (b) 3 (c) 4 (d) 7
18	The distance between the point (– 6 , 8) and the y-axis equals .... units. (a) 6 (b) – 6 (c) 8 (d) – 8
19	The distance between the point (– 5 , – 2) and the y-axis = ... length unit. (a) – 5 (b) – 2 (c) 2 (d) 5
20	The distance between the point (– 3 , – 4) and the y-axis equals .... length units. (a) 4 (b) – 4 (c) 3 (d) – 3
21	The distance between the two points (1 , – 1) and (4 , 3) equals .... length unit. (a) 3 (b) 4 (c) 5 (d) 7
22	The length of the line segment which is drawn between the two points (–1 , 4) , (5 , 12) equals .... length units. (a) 5 (b) 10 (c) 12 (d) 13
23	The distance between the two points (2 , 0) and (5 , 0) equals .... length units. (a) 3 (b) 4 (c) 5 (d) 6
24	If A (2 , – 1) and B (5 , 3) , then AB = ... (a) 15 (b) 5 (c) 3 (d) 2
25	The distance between the two points (3 , 0) and (0 , – 4) = ... length units. (a) 4 (b) 5 (c) 6 (d) 7
26	The distance between the two points (4 , 0) and (– 3 , 0) equals .... length units. (a) 5 (b) 7 (c) 1 (d) 4

27	The distance between the two points (7 , 4) , (3 , 1) equals …………… length unit. (a) 7 (b) 5 (c) 3 (d) 1
28	The distance between the two points (3 , a) and (– 1 , a) is …………… length unit. (a) 16 (b) 9 (c) 5 (d) 4
29	If the distance between the two points (a , 0) , (0 , 1) is 1 length unit , then a = …………… (a) – 1 (b) 0 (c) 1 (d) ± 1
30	The perpendicular distance between the two straight lines : $y + 1 = 0$, $y + 3 = 0$ equals …………… length unit. (a) 4 (b) 2 (c) 1 (d) 5
31	The perpendicular distance between the two straight lines : $x - 2 = 0$, $x + 3 = 0$ equals …………… length units. (a) 1 (b) 5 (c) 2 (d) 3
32	The perpendicular distance between the two straight lines : $y - 3 = 0$, $y + 2 = 0$ equals …………… length units. (a) 1 (b) 2 (c) 5 (d) 3
33	The perpendicular distance between the two straight lines : $y - 5 = 0$, $y + 6 = 0$ equals …………… length unit. (a) 1 (b) 5 (c) 11 (d) 6
34	The points (0 , 0) , (3 , 0) and (0 , 4) …………… (a) form an obtuse-angled triangle. (b) form an acute-angled triangle. (c) form a right-angled triangle. (d) are collinear.
35	In the square ABCD , if A (2 , – 5) , B (– 1 , – 1) , then the perimeter of the square is …………… length unit. (a) $4\sqrt{7}$ (b) 20 (c) 7 (d) 28
36	A circle of centre at the origin point and its radius length is 2 length unit , which of the following points belongs to the circle ? (a) (1 , – 2) (b) $\left(-2, \sqrt{5}\right)$ (c) $\left(\sqrt{3}, 1\right)$ (d) (0 , 1)
37	The circumference of the circle whose center is the origin point (0 , 0) and passes through the point (3 , 4) is …………… length unit. (a) 5π (b) 10π (c) 25π (d) 7π

38	**In the opposite figure :** ABCO is a rectangle , B (9 , 12) , then the length of $\overline{AC}$ equals .... units. (a) 9 (b) 12 (c) 13 (d) 15

[B] : Essay Problems : -

1	Prove that the points A (– 1 , – 4) , B (0 , 0) and C (2 , 8) are collinear. 2017 Exam (5) Question (2) (b)
2	**Prove that :** the points A (– 2 , 5) , B (3 , 3) and C (– 4 , 2) are non-collinear. 2018 Exam (14) Question (3) (b)
3	Determine the type of the triangle whose vertices are A (3 , 1) , B (– 1 , 4) and C (– 5 , 1) with respect to the lengths of its sides , then find its perimeter. 2017 Exam (1) Question (4) (b)
4	**State the type of the triangle LMN with respect to its side lengths where :** L (– 2 , 4) , M (3 , – 1) and N (4 , 5) 2018 Exam (2) Question (5) (b)
5	**Prove that :** the points A (– 3 , 0) , B (3 , 4) and C (1 , – 6) are the vertices of an isosceles triangle its vertex is A 2018 Exam (17) Question (2) (a)
6	**Prove that :** the points A (– 2 , 5) , B (3 , 3) , C (– 4 , 2) and D (– 9 , 4) are vertices of a parallelogram. 2018 Exam (11) Question (2) (b)
7	ABCD is a quadrilateral where : A (2 , 4) , B (– 3 , 0) , C (– 7 , 5) and D (– 2 , 9) (1) **Prove that :** the figure ABCD is a square. (2) **Find :** the area of the figure ABCD 2018 Exam (9) Question (5) (a)
8	**Prove that :** the triangle whose vertices are the points A (1 , 4) , B (– 1 , – 2) and C (2 , – 3) is right-angled at B 2018 Exam (16) Question (3) (a)
9	Prove that the points A (6 , 0) , B (2 , – 4) and C (– 4 , 2) are the vertices of a right-angled triangle at B 2017 Exam (15) Question (3) (b)

10	**Prove that :** The points A (3 , – 1) , B (– 4 , 6) and C (2 , – 2) are located on a circle whose centre is the point M (– 1 , 2) , **then find :** (1) The circumference of this circle. (2) The area of this circle. (where $\pi = 3.14$) 2018 Exam (6) Question (5) (b)
11	If the distance between the two points (a , 7) and (0 , 3) equals 5 length units. **Find :** the value of a 2018 Exam (10) Question (5) (a)
12	If the distance of the point (x , 5) from the point (6 , 1) equals $2\sqrt{5}$, then find the value of x 2018 Exam (13) Question (3) (a)
13	If the points X (3 , 5) , Y (4 , 2) and Z (–5 , a) are the vertices of a right-angled triangle at Y , find the value of a 2017 Exam (14) Question (4) (b)
14	ABCD is a parallelogram in which A = (x , 2) , B = (3 , 8) , C = (9 , 10) and D = (7 , 4) Find the value of x 2017 Exam (19) Question (3) (b)
15	**Prove that :** the points A (4 , 3) , B (1 , 1) and C (– 5 , – 3) are collinear. 2018 Exam (15) Question (2) (b)
16	**Prove that :** the points A (– 3 , – 1) , B (6 , 5) and C (3 , 3) are collinear. 2017 Exam (1) Question (2) (b)
17	Show the type of the triangle whose vertices are A (3 , 3) , B (1 , 5) and C (1 , 3) due to its side lengths. 2017 Exam (2) Question (2) (b)
18	Prove that the triangle whose vertices are A (1 , 1) , B (5 , 1) and C (3 , 4) is an isosceles triangle. 2017 Exam (14) Question (2) (b)
19	Prove that the points A (4 , 3) , B (7 , 0) , C (1 , – 2) are the vertices of a scalene triangle. 2017 Exam (3) Question (2) (b)
20	If the points A (– 1 , 3) , B (5 , 1) , C (6 , 4) and D (0 , 6) in the coordinates plane. **Prove that :** ABCD is a rectangle. 2018 Exam (19) Question (5) (b)
21	**Prove that :** Δ ABC in which A (1 , 1) , B (0 , 4) and C (– 1 , 1) is an isosceles triangle. 2018 Exam (4) Question (4) (b)

Prep. [3]

First Term

Geometry

Unit [5]

Lesson [2]

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Lesson [2] : Two Coordinates Of Midpoint Of A Line Segment

First point : A (X_1 , y_1) , Second point : A (X_2 , y_2)

Midpoint point : M (m_x , m_y) then

$M(m_x, m_y) = \left(\frac{X_1 + X_2}{2}, \frac{y_1 + y_2}{2}\right)$,

$X_1 = m_x \times 2 - X_2$ $y_1 = m_y \times 2 - y_2$

For Example : -

- If A (1 , 5) , B (3 , 1) and M is the midpoint of $\overline{AB}$, then :

$M = \left(\frac{1+3}{2}, \frac{5+1}{2}\right) = (2, 3)$

- If X (3 , – 2) , Y (– 1 , – 4) and M is the midpoint of $\overline{XY}$, then :

$M = \left(\frac{3+(-1)}{2}, \frac{-2+(-4)}{2}\right) = (1, -3)$

Remark : -

If $\overline{AB}$ is a diameter in a circle of centre M , then M is the midpoint of $\overline{AB}$

Exercises

[A] : Choose The Correct Answer : -

1	If A (3 , 4) , B (5 , 6) , then the coordinates of the midpoint of $\overline{AB}$ is …………… (a) (3 , 5) (b) (3 , 6) (c) (4 , 5) (d) (4 , 6)
2	If A (0 , – 2) , B (6 , 2) , then the midpoint of $\overline{AB}$ is …………… (a) (6 , 0) (b) (3 , 2) (c) (3 , 1) (d) (3 , 0)
3	If (4 , – 3) is the midpoint of $\overline{AB}$, where A (3 , – 4) , then the point B = …………… (a) (5 , – 2) (b) (2 , 5) (c) (5 , 2) (d) (3.5 , – 3.5)
4	The distance between the two points (0 , 0) , (3 , – 4) equals …………… length units. (a) 1 (b) 5 (c) – 1 (d) 7
5	The distance between the point (1 , 5) and the x-axis is …………… length unit. (a) 1 (b) 4 (c) 5 (d) 6

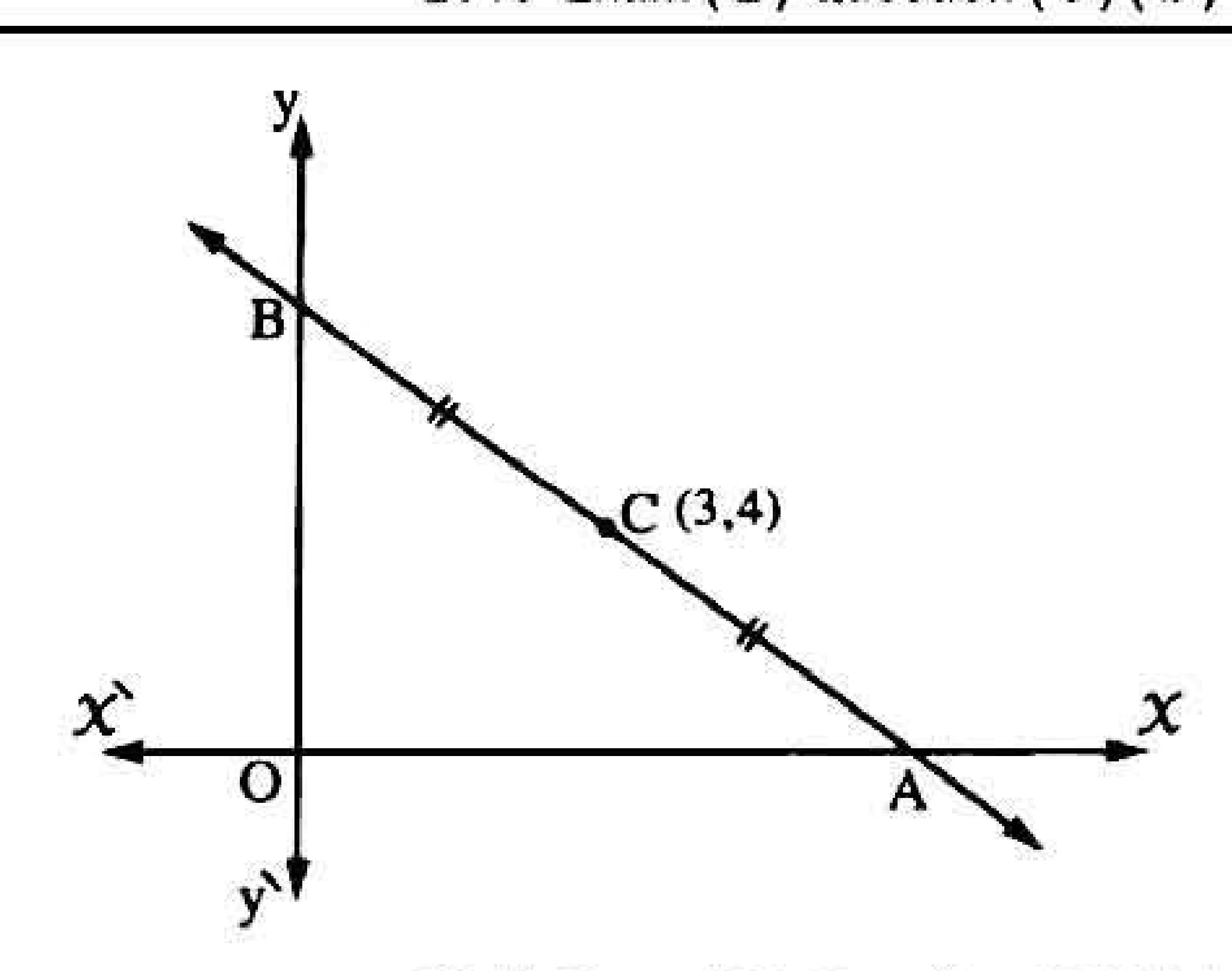
6	The distance between the point (3 , 4) and the y-axis is .... length unit. (a) 5 (b) 3 (c) 4 (d) 7	
7	The distance between the two points (2 , 0) and (5 , 0) equals .... length units. (a) 3 (b) 4 (c) 5 (d) 6	
8	The perpendicular distance between the two straight lines : y + 1 = 0 , y + 3 = 0 equals .... length unit. (a) 4 (b) 2 (c) 1 (d) 5	
9	The midpoint of $\overline{AB}$ where A (– 2 , 5) and B (– 2 , 3) is .... (a) (4 , – 2) (b) (2 , – 4) (c) (– 2 , 4) (d) (0 , 1)	
10	If the point (0 , 4) is the midpoint of the distance between the two points (– 1 , – 1) , (x , y) , then the point (x , y) is .... (a) (1 , 9) (b) (– 1 , 9) (c) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (d) (– 1 , 3)	
11	The distance between the two points (0 , 0) , (2 , 3) equals .... length unit. (a) $\sqrt{5}$ (b) $\sqrt{13}$ (c) $\sqrt{7}$ (d) $\sqrt{11}$	
12	The length of the line segment which is drawn between the two points (0 , 0) , (5 , 12) equals .... length unit. (a) 5 (b) 7 (c) 12 (d) 13	
13	The distance between the point (5 , – 2) and the x-axis equals .... length unit. (a) – 2 (b) 2 (c) 3 (d) 7	
14	The length of the line segment which is drawn between the two points (–1 , 4) , (5 , 12) equals .... length units. (a) 5 (b) 10 (c) 12 (d) 13	
15	If the distance between the two points (a , 0) , (0 , 1) is 1 length unit , then a = ... (a) – 1 (b) 0 (c) 1 (d) ± 1	
16	If A (– 1 , 2) , B (5 , – 10) , then the midpoint of $\overline{AB}$ is .... (a) (– 4 , – 2) (b) (– 2 , 4) (c) (2 , – 4) (d) (2 , 4)	
17	If the point (x , y) is the midpoint of the distance between the two points (– 1 , – 1) , (1 , 9) , then the point (x , y) is .... (a) (– 1 , 9) (b) (0 , 4) (c) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (d) (– 1 , 3)	

18	**In the opposite figure :** C (3 , 4) is the midpoint of $\overline{AB}$, then OA = ... length units. (a) 3 (b) 4 (c) 6 (d) 8
19	The distance between the point (4 , 3) and the origin point = ... length unit. (a) 1 (b) 3 (c) 4 (d) 5
20	The distance between the point (5 , $\tan^2 60°$) and the x-axis = ... length unit. (a) 3 (b) $\sqrt{5}$ (c) 3 (d) $\sqrt{3}$
21	The distance between the two points (1 , – 1) and (4 , 3) equals .... length unit. (a) 3 (b) 4 (c) 5 (d) 7
22	The distance between the two points (3 , a) and (– 1 , a) is .... length unit. (a) 16 (b) 9 (c) 5 (d) 4
23	The coordinates of the midpoint of the line segment $\overline{AB}$ where A (3 , 1) and B (– 1 , 3) is .... (a) (4 , – 2) (b) (2 , – 1) (c) (2 , 4) (d) (1 , 2)
24	If $\overline{AB}$ is a diameter of a circle , where A (3 , – 5) and B (5 , 1) , then the centre of the circle is .... (a) (4 , – 2) (b) (4 , 2) (c) (2 , 2) (d) (8 , – 2)
25	If the x-axis bisects $\overline{AB}$ such that : A (3 , 2) , B (– 2 , y) , then y = ... (a) 3 (b) 2 (c) – 2 (d) 4
26	The distance between the point (4 , – 3) and the origin point equals .... length units. (a) – 3 (b) 3 (c) 4 (d) 5
27	The distance between the point (4 , – 3) and the x-axis = ... length unit. (a) – 3 (b) 3 (c) 4 (d) 5
28	The distance between the point (– 3 , – 4) and the y-axis equals .... length units. (a) 4 (b) – 4 (c) 3 (d) – 3
29	The distance between the two points (7 , 4) , (3 , 1) equals .... length unit. (a) 7 (b) 5 (c) 3 (d) 1

30	The points (0 , 0) , (3 , 0) and (0 , 4) …………… (a) form an obtuse-angled triangle. (b) form an acute-angled triangle. (c) form a right-angled triangle. (d) are collinear.
31	If A (5 , 7) , B (1 , – 1) , then the midpoint of $\overline{AB}$ is …………… (a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)
32	$\overline{AB}$ is a diameter of a circle M , where A (– 2 , 3) , B (6 , – 5) , then M = …………… (a) (4 , 4) (b) (– 2 , 1) (c) (2 , – 1) (d) (– 1 , 2)
33	If C is the midpoint of $\overline{AB}$ where A (2 , 3) , B (6 , y) and C (4 , 6) , then y = …………… (a) 5 (b) 7 (c) 9 (d) 12
34	The length of the line segment which is drawn between the two points (0 , 0) , (– 4 , 3) equals …………… length unit. (a) 3 (b) 4 (c) $\sqrt{7}$ (d) 5
35	The distance between the point (3 , – 4) and the x-axis is …………… length unit. (a) – 3 (b) 4 (c) – 4 (d) 3
36	The distance between the point (– 5 , – 2) and the y-axis = …………… length unit. (a) – 5 (b) – 2 (c) 2 (d) 5
37	The distance between the two points (4 , 0) and (– 3 , 0) equals …………… length units. (a) 5 (b) 7 (c) 1 (d) 4
38	The perpendicular distance between the two straight lines : y – 5 = 0 , y + 6 = 0 equals …………… length unit. (a) 1 (b) 5 (c) 11 (d) 6
39	If A (3 , 4) and B (3 , 0) , then the coordinates of the midpoint of $\overline{AB}$ is …………… (a) (0 , – 2) (b) (6 , 4) (c) (3 , 2) (d) (3 , – 2)
40	If $\overline{AB}$ is a diameter of a circle where A (– 1 , 5) and B (3 , 1) , then the centre of this circle is …………… (a) (2 , 6) (b) (1 , 3) (c) (4 , – 4) (d) (– 4 , 4)
41	If (3 , – 1) is the midpoint of $\overline{AB}$ where A (x , 2) , B (– 1 , – 4) , then x = …………… (a) 17 (b) 6 (c) 13 (d) 7
42	If O (0 , 0) and A (3 , 4) , then the length of $\overline{OA}$ = …………… length unit. (a) 3 (b) 4 (c) 5 (d) 7

43	The point (2 , – 4) lies at distance from x-axis = ... length unit. (a) 4 (b) 2 (c) – 4 (d) 6
44	The distance between the point (– 6 , 8) and the y-axis equals .... units. (a) 6 (b) – 6 (c) 8 (d) – 8
45	The distance between the two points (3 , 0) and (0 , – 4) = ... length units. (a) 4 (b) 5 (c) 6 (d) 7
46	The perpendicular distance between the two straight lines : y – 3 = 0 , y + 2 = 0 equals .... length units. (a) 1 (b) 2 (c) 5 (d) 3
47	If $\overline{AB}$ is a diameter in a circle where A (3 , 4) , B (3 , 0) , then the point of the centre of the circle is .... (a) (3 , 4) (b) (3 , 2) (c) (3 , – 2) (d) (0 , – 2)
48	If the point (2 , 1) is the midpoint of the distance between the points (3 , – 4) and (x , 6) , then x = ... (a) 3 (b) 6 (c) – 1 (d) 1
49	The distance between the point (– 3 , 4) and the origin point on a perpendicular coordinate plane = ... length unit. (a) – 5 (b) 25 (c) $\frac{1}{5}$ (d) 5
50	The distance between the point (4 , 3) and x-axis is .... (a) – 3 (b) 3 (c) 4 (d) – 4
51	The distance between the point (– 3 , 4) and y-axis equals .... length units. (a) 1 (b) 3 (c) 4 (d) 7
52	If A (2 , – 1) and B (5 , 3) , then AB = ... (a) 15 (b) 5 (c) 3 (d) 2
53	The perpendicular distance between the two straight lines : x – 2 = 0 , x + 3 = 0 equals .... length units. (a) 1 (b) 5 (c) 2 (d) 3

[B] : Essay Problems : -

1	Find the coordinates of the midpoint of $\overline{AB}$ where A (2 , 4) , B (6 , 0) 2018 Exam (20) Question (2) (a)
2	If the point C (x , – 3) is the midpoint of $\overline{AB}$ where A (– 3 , y) and B (9 , – 7) , then find the value of each of x and y 2017 Exam (1) Question (2) (b)
3	If C is the midpoint of $\overline{AB}$, A = (x , 7) , B = (1 , y) , C = (2 , 2) **, find the value of each of :** x , y 2017 Exam (2) Question (5) (a)
4	If the point (3 , 1) is the midpoint of (1 , y) , (x , 3) , find the point (x , y) 2017 Exam (2) Question (3) (b)
5	If A (– 1 , – 1) , B (2 , 3) , C (6 , 0) and D (3 , – 4) are four points on an orthogonal Cartesian coordinates plane. **Prove that :** $\overline{AC}$ and $\overline{BD}$ bisect each other. What is the name of the figure ABCD ? 2018 Exam (16) Question (5) (a)
6	$\overline{AB}$ is a diameter of a circle M If B = (8 , 11) , M = (5 , 7) , find the coordinates of A and find the circumference of the circle. 2017 Exam (4) Question (5) (a)
7	Prove that the points A (– 3 , 0) , B (3 , 4) , C (1 , – 6) are the vertices of an isosceles triangle of vertex A , then find the length of the drawn line segment from A perpendicular to $\overline{BC}$ 2017 Exam (6) Question (4) (b)
8	ABCD is a parallelogram , its two diagonals intersect at E where : A (3 , – 1) , B (6 , 2) and C (1 , 7) Find the coordinates of the points E and D 2018 Exam (2) Question (3) (b)
9	**In the opposite figure :** C (3 , 4) is the midpoint of $\overline{AB}$ **Find :** The perimeter of triangle OAB  2018 Exam (3) Question (5) (b)

Prep. [3]

First Term

Geometry

Unit [5]

Lesson [3]

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Lesson [3] : The Slope Of The Straight Line

Prelude

You studied before the slope of the straight line given two points on it.

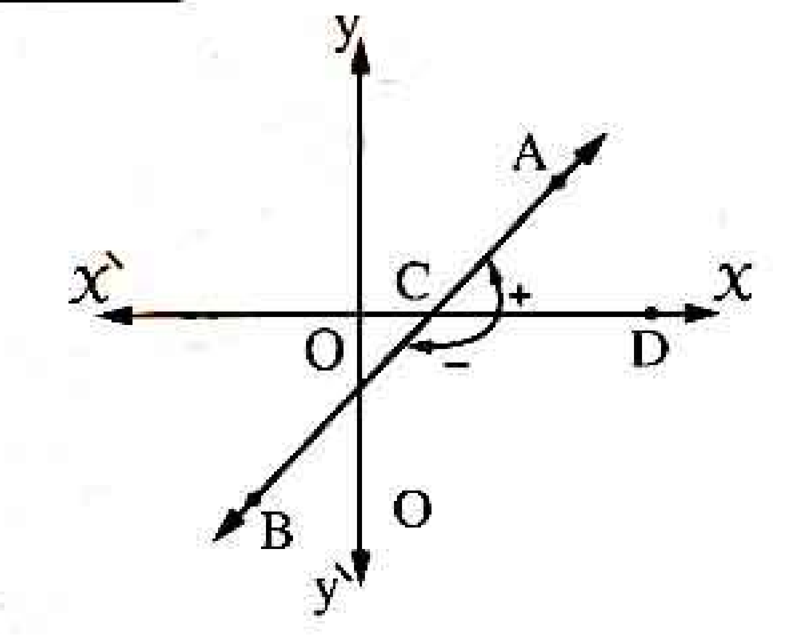
If A and B are two points in the coordinates plane where A (x_1 , y_1) and B (x_2 , y_2) , then :

The slope of the straight line $\overleftrightarrow{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_1 \neq x_2$

The positive measure and the negative measure of an angle

In the opposite figure :

If $\overleftrightarrow{AB}$ intersects the x-axis at the point C , then $\overleftrightarrow{AB}$ makes two angles with the positive direction of the x-axis.



The slope of the straight line

Definition

The slope of the straight line is the tangent of the positive angle which this straight line makes with the positive direction of the x-axis.

i.e. The slope of the straight line = tan θ where θ is the measure of the positive angle which the straight line makes with the positive direction of the x-axis.

Notice that

The straight line passes through the two points (2 , 0) and (7 , 5) , then :

the slope of the straight line L $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{7 - 2} = \frac{5}{5} = 1$

Remark

The angle which the straight line L makes with the positive direction of the x-axis takes one of the following cases :

1 Acute angle	2 Obtuse angle	3 Zero angle	4 Right angle
The slope is positive	The slope is negative	The slope is zero	The slope is undefined

The relation between the two slopes of the two parallel straight lines

Also , we can deduce the opposite :

If $\boxed{m_1 = m_2}$, then $\boxed{L_1 // L_2}$

i.e. If the two straight lines have equal slopes , then the two straight lines are parallel.

The relation between the slopes of the two perpendicular (orthogonal) straight lines

If L_1 and L_2 are two straight lines of slopes m_1 and m_2 respectively and $\boxed{L_1 \perp L_2}$, then $\boxed{m_1 \times m_2 = -1}$, unless one of them is parallel to one of the coordinate axes.

i.e. The product of the slopes of the perpendicular straight lines = – 1

and vice versa :

Remark

If $L_1 \perp L_2$, the slope of L_1 is m_1 and the slope of L_2 is m_2 , then $m_2 = \frac{-1}{m_1}$, $m_1 = \frac{-1}{m_2}$

For example :

- If the slope of the straight line L is 2 , then the slope of the perpendicular to it $= -\frac{1}{2}$
- If the slope of the straight line L is $-\frac{2}{3}$, then the slope of the perpendicular to it $= \frac{3}{2}$

Remarks to solve the problems on quadrilateral

- **To prove that a quadrilateral is a trapezium , we prove that :**
  Two opposite sides are parallel and the other two sides are not parallel.
- **To prove that a quadrilateral is a parallelogram , we prove only one of the following properties :**
  1. Each two opposite sides are parallel.
  2. Each two opposite sides are equal in length.
  3. Two opposite sides are parallel and equal in length.
  4. The two diagonals bisect each other.
- **To prove that a quadrilateral is a rectangle , rhombus or square , we prove at first that the quadrilateral is a parallelogram , then :**

– To prove that the parallelogram is a rectangle , we prove only one of the following two properties :

1. Two adjacent sides are perpendicular.
2. The two diagonals are equal in length.

– To prove that the parallelogram is a rhombus , we prove only one of the following two properties :

1. Two adjacent sides are equal in length.
2. The two diagonals are perpendicular.

– To prove that the parallelogram is a square, we prove only one of the following properties :

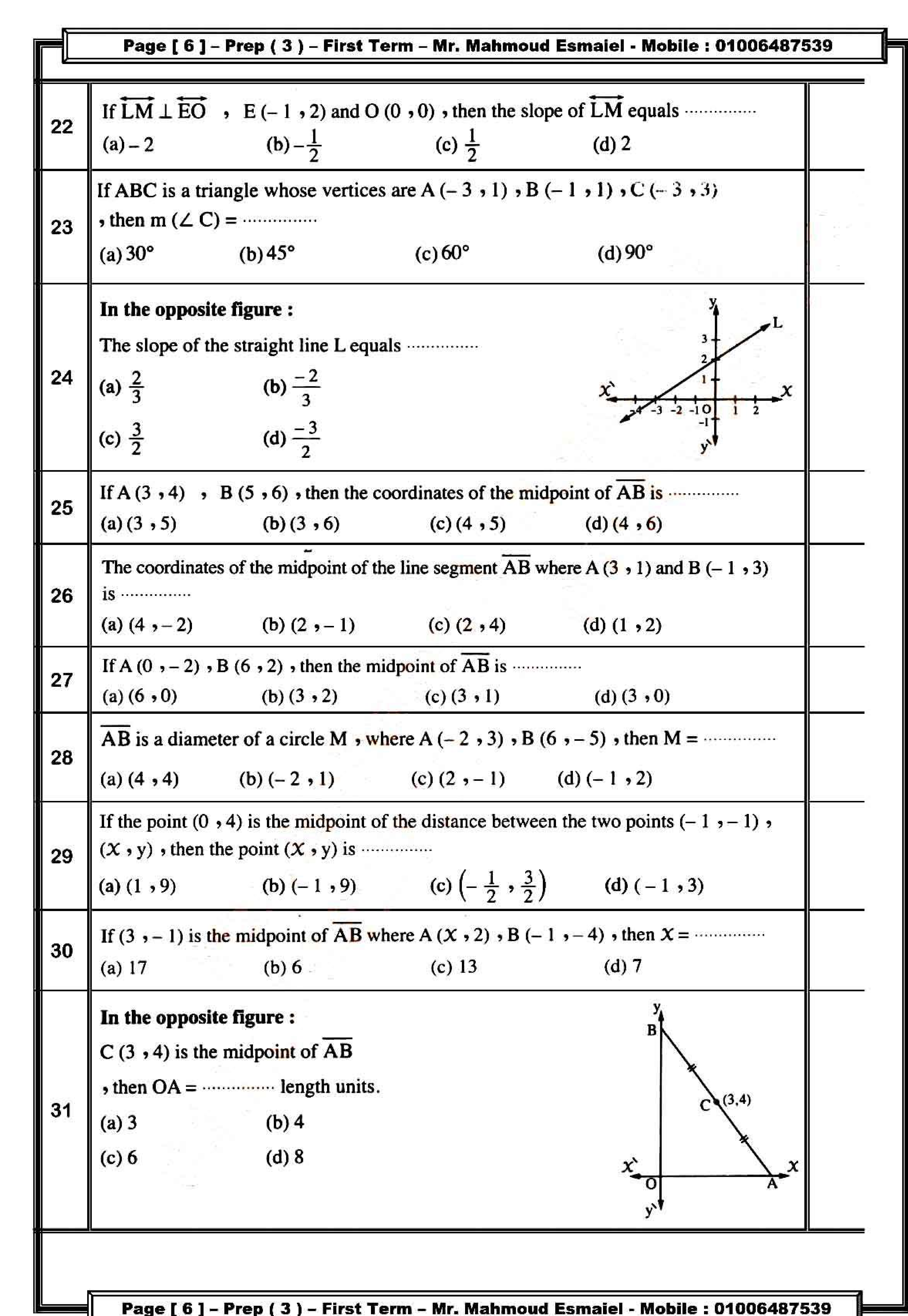
1. Two adjacent sides are perpendicular and equal in length.
2. Two adjacent sides are perpendicular and its diagonals are perpendicular.
3. Two diagonals are equal in length and perpendicular.
4. Two adjacent sides are equal in length and its two diagonals are equal in length.

Exercises

[A] : Choose The Correct Answer : -

1	The slope of the straight line that makes with the positive direction to the X-axis a positive angle of $\theta°$ measure = …………… (a) $\sin\theta$ (b) $\cos\theta$ (c) $\frac{\sin\theta}{\cos\theta}$ (d) $\sin\theta + \cos\theta$
2	The slope of the straight line which is parallel to the X-axis is …………… (a) – 1 (b) zero (c) 1 (d) undefined.
3	The slope of the straight line which is parallel to the y-axis is …………… (a) 0 (b) undefined. (c) – 1 (d) 1
4	If m_1 , m_2 are the slopes of two perpendicular straight lines , then $m_1 \times m_2$ = …………… (a) – 1 (b) zero (c) 1 (d) 2
5	If the slopes of two straight lines are equal , then the two lines are …………… (a) perpendicular. (b) parallel. (c) intersecting. (d) not parallel.
6	The slope of the straight line which makes an angle of measure 45° with the positive direction of the X-axis is …………… (a) zero (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{3}$
7	The slope of the straight line that makes a positive angle in the positive direction of X-axis its measure is 45° equals …………… (a) 1 (b) – 1 (c) zero (d) 2
8	In the parallelogram XYZL , the slope of $\overline{XY}$ is equal to the slope of …………… (a) $\overline{XL}$ (b) $\overline{XZ}$ (c) $\overline{YZ}$ (d) $\overline{LZ}$
9	If $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD}$ = …………… (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$

10	If $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = -2$, then the slope of $\overleftrightarrow{CD} =$ .... (a) -2 (b) $\frac{-1}{2}$ (c) $\frac{1}{2}$ (d) undefined.
11	If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB}$ = zero , then the slope of $\overleftrightarrow{CD}$ .... (a) $= -1$ (b) $= 1$ (c) $= 0$ (d) is undefined.
12	If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = 0.5$, then the slope of $\overleftrightarrow{DC} =$ .... (a) 1 (b) 2 (c) 0.5 (d) -2
13	If the slope of $\overleftrightarrow{AB} = \frac{1}{3}$ and $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, then the slope of $\overleftrightarrow{CD} =$ .... (a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) 3 (d) -3
14	Two perpendicular lines their slopes are $\frac{-1}{4}$ and 4 k , then k = ... (a) 4 (b) 1 (c) -4 (d) $\frac{1}{4}$
15	If $\frac{-2}{3}$, $\frac{k}{2}$ are the slopes of two parallel straight lines , then k = ... (a) 3 (b) $\frac{1}{3}$ (c) $\frac{-3}{4}$ (d) $\frac{-4}{3}$
16	If $\frac{2}{3}$, $\frac{k}{2}$ are the slopes of two parallel straight lines , then k = ... (a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $-\frac{2}{3}$ (d) 3
17	If $\frac{-3}{2}$, $\frac{6}{k}$ are the slopes of two parallel straight lines , then k = ... (a) 6 (b) -4 (c) $\frac{3}{2}$ (d) 2
18	The slope of straight line perpendicular to the straight line passes through the two points (2 , 3) and (5 , 1) = ... (a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{-3}{2}$ (d) $\frac{-2}{3}$
19	The slope of the straight line parallel to the straight line passing through the two points (3 , -2) , (-1 , 3) equals .... (a) $\frac{4}{5}$ (b) $\frac{5}{4}$ (c) $\frac{-5}{4}$ (d) $-\frac{4}{5}$
20	If the straight line $\overleftrightarrow{CD}$ is parallel to the y-axis where C (m , 4) , D (-5 , 7) , then m = ... (a) zero (b) -5 (c) 3 (d) 5
21	If the straight line $\overleftrightarrow{AB}$ // the x-axis where A (8 , 3) and B (2 , k) , then k = ... (a) 8 (b) 0 (c) 3 (d) 2



22 If $\overleftrightarrow{LM} \perp \overleftrightarrow{EO}$, E (– 1 , 2) and O (0 , 0) , then the slope of $\overleftrightarrow{LM}$ equals ……………

(a) – 2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2

23 If ABC is a triangle whose vertices are A (– 3 , 1) , B (– 1 , 1) , C (– 3 , 3) , then m (∠ C) = ……………

(a) 30° (b) 45° (c) 60° (d) 90°

24 **In the opposite figure :**

The slope of the straight line L equals ……………

(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$

(c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

25 If A (3 , 4) , B (5 , 6) , then the coordinates of the midpoint of $\overline{AB}$ is ……………

(a) (3 , 5) (b) (3 , 6) (c) (4 , 5) (d) (4 , 6)

26 The coordinates of the midpoint of the line segment $\overline{AB}$ where A (3 , 1) and B (– 1 , 3) is ……………

(a) (4 , – 2) (b) (2 , – 1) (c) (2 , 4) (d) (1 , 2)

27 If A (0 , – 2) , B (6 , 2) , then the midpoint of $\overline{AB}$ is ……………

(a) (6 , 0) (b) (3 , 2) (c) (3 , 1) (d) (3 , 0)

28 $\overline{AB}$ is a diameter of a circle M , where A (– 2 , 3) , B (6 , – 5) , then M = ……………

(a) (4 , 4) (b) (– 2 , 1) (c) (2 , – 1) (d) (– 1 , 2)

29 If the point (0 , 4) is the midpoint of the distance between the two points (– 1 , – 1) , (x , y) , then the point (x , y) is ……………

(a) (1 , 9) (b) (– 1 , 9) (c) $\left(-\frac{1}{2} , \frac{3}{2}\right)$ (d) (– 1 , 3)

30 If (3 , – 1) is the midpoint of $\overline{AB}$ where A (x , 2) , B (– 1 , – 4) , then x = ……………

(a) 17 (b) 6 (c) 13 (d) 7

31 **In the opposite figure :**

C (3 , 4) is the midpoint of $\overline{AB}$

, then OA = …………… length units.

(a) 3 (b) 4

(c) 6 (d) 8

32	If A (3 , 4) and B (3 , 0) , then the coordinates of the midpoint of $\overline{AB}$ is .... (a) (0 , – 2) (b) (6 , 4) (c) (3 , 2) (d) (3 , – 2)
33	In the square ABCD , if A (2 , – 5) , B (– 1 , – 1) , then the perimeter of the square is .... length unit. (a) $4\sqrt{7}$ (b) 20 (c) 7 (d) 28
34	A circle of centre at the origin point and its radius length is 2 length unit , which of the following points belongs to the circle ? (a) (1 , – 2) (b) $\left(-2 , \sqrt{5}\right)$ (c) $\left(\sqrt{3} , 1\right)$ (d) (0 , 1)
35	The circumference of the circle whose center is the origin point (0 , 0) and passes through the point (3 , 4) is .... length unit. (a) 5 π (b) 10 π (c) 25 π (d) 7 π
36	**In the opposite figure :** ABCO is a rectangle , B (9 , 12) , then the length of $\overline{AC}$ equals .... units. (a) 9 (b) 12 (c) 13 (d) 15
37	The distance between the point (1 , 5) and the X-axis is .... length unit. (a) 1 (b) 4 (c) 5 (d) 6
38	The distance between the point (4 , 3) and X-axis is .... (a) – 3 (b) 3 (c) 4 (d) – 4
39	The point (2 , – 4) lies at distance from X-axis = ... length unit. (a) 4 (b) 2 (c) – 4 (d) 6
40	The distance between the point (3 , – 4) and the X-axis is .... length unit. (a) – 3 (b) 4 (c) – 4 (d) 3
41	The distance between the point (4 , – 3) and the X-axis = ... length unit. (a) – 3 (b) 3 (c) 4 (d) 5
42	The distance between the point (5 , $\tan^2 60°$) and the X-axis = ... length unit. (a) 3 (b) $\sqrt{5}$ (c) 3 (d) $\sqrt{3}$

43	If the distance between the two points (a , 0) , (0 , 1) is 1 length unit , then a = ... (a) – 1 (b) 0 (c) 1 (d) ± 1
44	The perpendicular distance between the two straight lines : y + 1 = 0 , y + 3 = 0 equals .... length unit. (a) 4 (b) 2 (c) 1 (d) 5
45	The perpendicular distance between the two straight lines : $x - 2 = 0$, $x + 3 = 0$ equals .... length units. (a) 1 (b) 5 (c) 2 (d) 3
46	The perpendicular distance between the two straight lines : y – 3 = 0 , y + 2 = 0 equals .... length units. (a) 1 (b) 2 (c) 5 (d) 3
47	

[B] : Essay Problems : -

1	**Prove that :** the points A (– 1 , 5) , B (1 , 2) and C (3 , – 1) are collinear. 2018 Exam (2) Question (4) (b)
2	Prove that the points A (– 1 , – 4) , B (0 , 0) and C (2 , 8) are collinear. 2017 Exam (5) Question (2) (b)
3	Find the slope of the perpendicular straight line to the line which passes through the two points (3 , – 2) and (5 , 1) 2018 Exam (20) Question (5) (a)
4	**Prove that :** the straight line which passes through the two points (3 , 1) and (2 , 2) is perpendicular to the straight line which makes a positive angle of measure 45° with the positive direction of x-axis. 2018 Exam (15) Question (4) (b)
5	**Prove that :** Δ ABC in which A (1 , 1) , B (0 , 4) and C (– 1 , 1) is an isosceles triangle. 2018 Exam (4) Question (4) (b)
6	**Prove that :** triangle whose vertices are Y (4 , 2) , X (3 , 5) and Z (– 5 , – 1) is right-angled at Y 2018 Exam (18) Question (4) (a)
7	Using the slope to prove that the points : A (6 , 0) , B (2 , – 4) and C (– 4 , 2) are vertices of a right-angled triangle at B , then find the coordinates of the point D which make the figure ABCD a rectangle. 2018 Exam (6) Question (4) (b)
8	2017 Exam (17) Question (5) (a)

	Prove that the points A (– 1 , 1) , B (0 , 5) , C (4 , 2) , D (3 , – 2) are the vertices of a parallelogram.
9	**Using the slope prove that :** the points A (– 1 , 3) , B (5 , 1) , C (6 , 4) and D (0 , 6) are vertices of a rectangle. 2018 Exam (3) Question (4) (a)
10	ABCD is a quadrilateral where : A (2 , 4) , B (– 3 , 0) , C (– 7 , 5) and D (– 2 , 9) (1) **Prove that :** the figure ABCD is a square. (2) **Find :** the area of the figure ABCD 2018 Exam (9) Question (5) (a)
11	If the points A (6 , – 1) , B (m , 0) and C (– 4 , 4) are collinear , **find :** (1) The value of m (2) The equation of $\overleftrightarrow{AB}$ 2017 Exam (9) Question (4) (a)
12	If the triangle whose vertices are Y (4 , 2) , X (3 , 5) and Z (– 5 , a) is a right-angled at Y **Find** : the value of a 2018 Exam (6) Question (2) (b)
13	ABCD is a parallelogram in which A = (x , 2) , B = (3 , 8) , C = (9 , 10) and D = (7 , 4) Find the value of x 2017 Exam (19) Question (3) (b)
14	If the points X (3 , 5) , Y (4 , 2) and Z (–5 , a) are the vertices of a right-angled triangle at Y , find the value of a 2017 Exam (14) Question (4) (b)
15	If the axis of symmetry of $\overline{CD}$ is passing through the point A (6 , m) where C (3 , 1) , D (– 3 , 7) , then find the value of : m 2017 Exam (8) Question (5) (b)
16	**In the opposite figure :** O is the origin point , A , B , D ∈ x-axis , the slope of $\overleftrightarrow{BC} = \sqrt{3}$, the equation of $\overleftrightarrow{AC}$ is : $x - y = 3$ **Find :** (1) The slope of $\overleftrightarrow{AC}$ and the length of $\overline{OH}$ (2) m (∠ CBD) and m (∠ CAD) (3) m (∠ ACB) [Figure: y, C, x`, x, O, A, D, H] 2017 Exam (5) Question (5) (b)

Prep. [3]

First Term

Geometry

Unit [5]

Lesson [4]

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Lesson [4] : The Equation Of The Straight Line

First Finding the slope of a straight line and the length of the intercepted part from y-axis.

If the equation of a straight line in the form : $y = m\,x + c$, then :

- The slope of the straight line = m
- The length of the intercepted part from y-axis = $|c|$
  and it passes through the point (0 , c)

For Example : -

- The straight line whose equation is $y = \frac{1}{2}x + 7$

  its slope = $\frac{1}{2}$

  and the intercepted part from y-axis = 7 length units and passes through the point (0 , 7)

- The straight line whose equation is $y = 3x - 5$, its slope = 3

  and cuts from the negative side of y-axis a part of 5 length units and passes through the point (0 , – 5)

Remarks

If the equation of a straight line in the form : $a\,x + b\,y + c = 0$

, then the slope of the straight line = $\frac{-\text{ coefficient of } x}{\text{coefficient of } y}$

and the straight line cuts y-axis at the point $\left(0 , \frac{-c}{b}\right)$

i.e. The length of the intercepted part from y-axis = $\left|\frac{-c}{b}\right|$

For Example : -

1 The straight line whose equation : $x - 2y + 3 = 0$

Its slope = $\frac{-1}{-2} = \frac{1}{2}$ and cut y-axis at the point $\left(0 , \frac{3}{2}\right)$

i.e. The straight line intercepts a part of length equals $\frac{3}{2}$ length unit from the positive side of y-axis.

2 The straight line whose equation : $3x + y + 4 = 0$

Its slope = – 3 and cut y-axis at the point (0 , – 4)

i.e. The straight line intercepts a part of length equals 4 length units from the negative side of y-axis.

Second Finding the equation of the straight line given its slope and the length of intercepted part of y-axis

The straight line whose slope = m and cuts y-axis at the point (0 , c) its equation is in the form :

$y = m x + c$

1 The equation of the straight line which passes through the origin point O (0 , 0) is $y = m x$, where m is the slope of the straight line.

2 The equation of x-axis is $y = 0$

3 The equation of y-axis is $x = 0$

4 The equation of the straight line parallel to x-axis and passes through the point $(0 , \ell)$ is $y = \ell$

5 The equation of the straight line which is parallel to y-axis and passes through the point (k , 0) is $x = k$

Exercises

[A] : Choose The Correct Answer : -

1	The equation of the straight line whose slope is – 1 and passes through the origin point is .... (a) $x = 1$ (b) $y = 1$ (c) $x = y$ (d) $y = - x$
2	The equation of the straight line whose slope equals 1 and passes through the origin point is .... (a) $x = 1$ (b) $y = 1$ (c) $y = x$ (d) $y = - x$
3	The equation of the straight line whose slope = $\frac{1}{2}$ and intercepts from a positive part from the y-axis 3 units is .... (a) $y = 2x + 3$ (b) $y = x + 6$ (c) $2y + x = 6$ (d) $2y = x + 6$
4	The equation of the sraight line which passes through the point (3 , – 5) and parallel to y-axis is .... (a) $x = 3$ (b) $y = - 5$ (c) $y = 2$ (d) $x = - 5$
5	The equation of the straight line which passes through the point (3 , – 2) and is parallel to y-axis is .... (a) $x = 3$ (b) $y = - 2$ (c) $x = - 2$ (d) $y = 3$

6	The equation of the straight line which is parallel to x-axis and passes through the point (0 , 2) is .... (a) $y = -3$ (b) $x = 2$ (c) $x = -3$ (d) $y = 2$
7	The equation of the straight line which passes through the point (3 , 5) and is parallel to x-axis is .... (a) $y = 3$ (b) $x = 3$ (c) $x = 5$ (d) $y = 5$
8	The equation of the straight line which passes through the point (– 2 , – 3) and parallel to x-axis is .... (a) $x = -2$ (b) $x = -3$ (c) $y = -2$ (d) $y = -3$
9	The equation of the straight line passing through the point (2 , – 3) and parallel to the x-axis is .... (a) $x = 2$ (b) $y = 3$ (c) $x = -2$ (d) $y = -3$
10	The slope of the straight line $2y = \frac{1}{2}(3 - 5x)$ equals .... (a) $\frac{-5}{2}$ (b) $\frac{-5}{4}$ (c) $\frac{3}{4}$ (d) $\frac{3}{2}$
11	The slope of the line : $2y - 6x = 5$ equals .... (a) 6 (b) – 6 (c) $\frac{5}{3}$ (d) 3
12	The slope of the straight line whose equation is : $3y = 2x - 5$ is .... (a) 3 (b) 2 (c) $\frac{2}{3}$ (d) – 5
13	The slope of the straight line whose equation is : $2x - 3y + 5 = 0$ equals .... (a) $\frac{-3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-2}{3}$
14	The slope of the straight line whose equation is : $3x - 4y + 12 = 0$ is .... (a) $\frac{3}{4}$ (b) $\frac{-3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{-4}{3}$
15	The straight line whose equation is : $3x + 4y - 9 = 0$, is perpendicular to the straight line whose slope is .... (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{-4}{3}$ (d) $\frac{-3}{4}$
16	The straight line whose equation is : $y - 2x - 5 = 0$ intercepts from y-axis a part of length .... units. (a) 2 (b) 5 (c) 7 (d) 10
17	The line : $2x - 3y - 6 = 0$ cuts a part of the y-axis of length .... units. (a) – 6 (b) – 2 (c) $\frac{2}{3}$ (d) 2

18	The line whose equation is : $2x - 3y = 6$ intercepts a part of y-axis of length …………… units. (a) – 6 (b) – 2 (c) 9 (d) 2	
19	The straight line whose equation is : $2x + 3y - 6 = 0$ intercepts from the y-axis a part of length …………… units. (a) – 6 (b) – 2 (c) $\frac{2}{3}$ (d) 2	
20	The straight line whose equation is : $3y = 2x + 6$ cuts from y-axis a part of length = …………… unit. (a) 6 (b) 3 (c) 2 (d) $\frac{2}{3}$	
21	The straight line whose equation is : $3y = 4x - 12$ intercepts from the y-axis a part of length …………… units. (a) $\frac{4}{3}$ (b) 3 (c) 4 (d) – 4	
22	If the two straight lines $x + y = 5$ and $kx + 2y = 0$ are parallel , then k = …………… (a) – 2 (b) – 1 (c) 1 (d) 2	
23	If the straight line : $Lx - 5y + 7 = 0$ is parallel to x-axis , then L = …………… (a) zero (b) 1 (c) 5 (d) 7	
24	If $x + y = 5$, $kx + 2y = 0$ are parallel , then k = …………… (a) – 2 (b) – 1 (c) 1 (d) 2	
25	If $x + y = 5$, $kx + 2y = 0$ are perpendicular , then k = …………… (a) – 2 (b) – 1 (c) 1 (d) 2	
26	The two straight lines $L_1 : y = ax + b$, $L_2 : y = cx + d$ are both perpendicular , then …………… = – 1 (a) b d (b) a c (c) a d (d) b c	
27	If the two straight lines $3x - 4y - 3 = 0$, $ky + 4x - 8 = 0$ are perpendicular , then k = …………… (a) 3 (b) 4 (c) – 3 (d) – 4	
28	If the straight line $y = x \sin 30° + c$ passes through the point (4 , 6) , then c = …………… (a) 4 (b) 6 (c) 8 (d) 2	

29	If the straight line whose equation is $x + 2y - 3 = 0$ is perpendicular to the straight line whose equation is $kx - 2y + 1 = 0$, then k = ... (a) 4 (b) 3 (c) 1 (d) 2
30	If the two straight lines : $x + y = 5$, $kx + 2y =$ zero are parallel , then k = ... (a) – 2 (b) – 1 (c) 1 (d) 2
31	If the two straight lines $2x + by + 3 = 0$, $3x - y + 2 = 0$ are perpendicular , then b = ... (a) 6 (b) 5 (c) 3 (d) 2
32	If the point (0 , a) belongs to the straight line : $3x - 4y + 12 = 0$, then a = ... (a) $\frac{1}{3}$ (b) – 3 (c) 4 (d) 3
33	If the two straight lines : $3x - 4y - 3 = 0$, $kx + 3y - 8 = 0$ are perpendicular , then k = ... (a) 4 (b) – 4 (c) 3 (d) – 3
34	The straight line of the equation $y = x$ makes an angle of measure .... with the positive direction of x-axis. (a) 45° (b) 60° (c) 90° (d) 135°
35	The straight line which its slope is the additive neutral is parallel to the straight line which its equation is .... (a) $y = x$ (b) $y = 1$ (c) $x = 1$ (d) $y = -x$
36	The straight line whose equation is $x = 3$ passes through the point .... (a) (1 , 3) (b) (4 , 3) (c) (3 , 5) (d) (0 , 3)
37	**In the opposite figure :** The length of the part which the straight line cuts from the y-axis equals .... unit of length. (a) – 3 (b) – 2 (c) 2 (d) 3 Figure labels: y, x\`, x, y\`, 1, -1, O, 1, 2, 3, -1, -2, -3
38	**In the opposite figure :** The equation of the straight line L is .... (a) $x = 1$ (b) $y = 1$ (c) $y = x$ (d) $y = -x$ Figure labels: y, L, x\`, 45°, x, O, y\`

39

In the opposite figure :

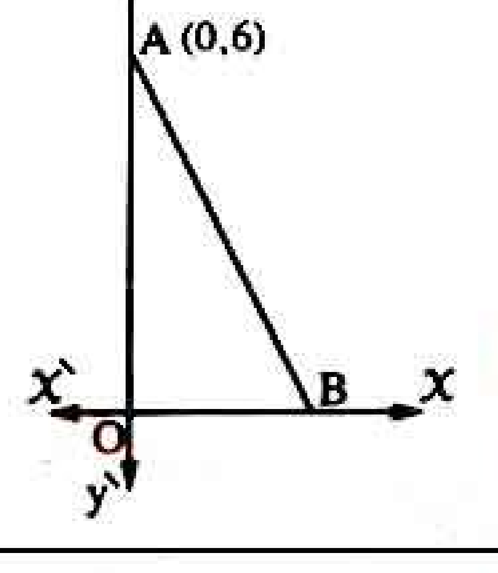
If the area of Δ AOB = 9 square units

, then the equation of the straight line $\overleftrightarrow{AB}$ = ...



(a) $y = 2x + 6$ (b) $y = 6 - 2x$

(c) $y = 2x - 6$ (d) $y = \frac{1}{2}x - 6$



40

In the opposite figure :

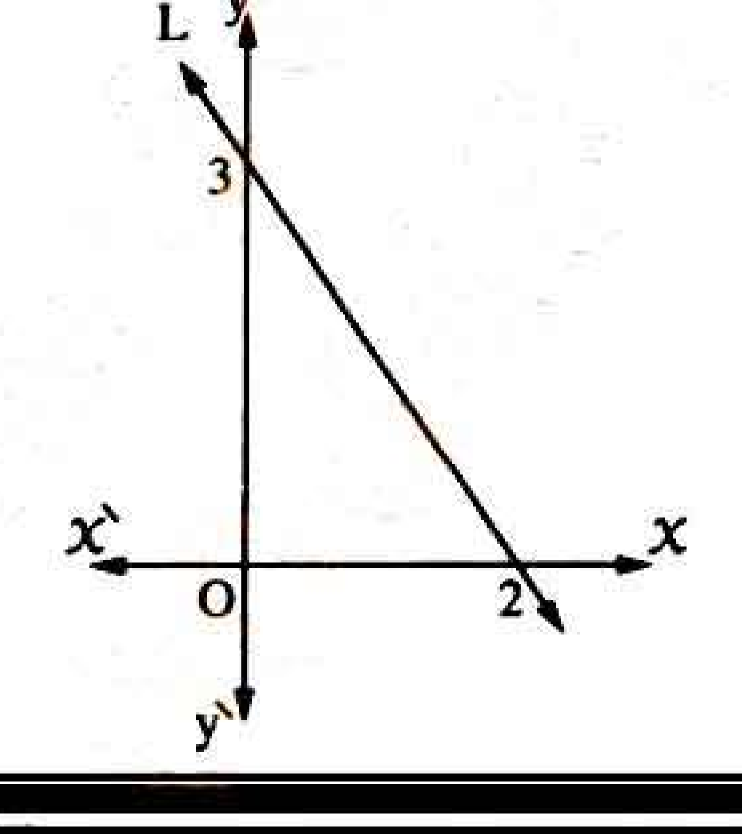
The equation of L is ....

(a) $y = 2x + 3$

(b) $2x + 3y = 0$

(c) $\frac{x}{2} + \frac{y}{3} = 1$

(d) $\frac{x}{2} + \frac{y}{3} = 5$



[B] : Essay Problems : -

1 Find the equation of the straight line which passes through the point (3 , 2) and its slope equals $\frac{1}{3}$

2018 Exam (5) Question (3) (b)

2 Find the equation of the straight line passing through the point (3 , 4) and makes with the positive direction of x-axis an angle its measure is 45°

2018 Exam (9) Question (2) (a)

3 Find the equation of straight line which passes through the points (1 , 3) and (– 1 , – 3) and prove that it is passing through the origin point.

2017 Exam (2) Question (3) (a)

4 Find the equation of straight line which passes through the point (1 , 6) and the midpoint of $\overline{AB}$ where A (1 , – 2) and B (3 , – 4)

2018 Exam (19) Question (2) (a)

5 Find the equation of straight line which passes through the two points A (2 , 3) and B (3 , 2)

2018 Exam (1) Question (2) (b)

6 Find the equation of the straight line which intercepts the two axes two positive parts of lengths 1 and 4 for x and y axes respectively and find its slope.

2017 Exam (2) Question (4) (a)

7	Find the equation of the straight line passing through the point (0 , 2) and parallel to the straight line its slope is $-\frac{1}{3}$ 2018 Exam (17) Question (4) (a)
8	Find the equation of the straight line which passes through the point (1 , 6) and is parallel to the straight line that makes with the positive direction of the x-axis an angle of measure 45° 2017 Exam (11) Question (3) (b)
9	Find the equation of the straight line which passes through the point (3 , – 4) and is parallel to the straight line which passes through the two points (– 1 , 7) , (5 , 3) 2017 Exam (7) Question (3) (a)
10	Find the equation of the straight line passing through the point (1 , 2) and parallel to the straight line : $2x + y - 6 = 0$ 2018 Exam (10) Question (2) (b)
11	Find the equation of the straight line passing through the point (0 , 3) and perpendicular to the line : $2x + 3y = 5$ 2018 Exam (8) Question (4) (a)
12	Find the equation of the straight line passing through the point (1 , 3) and perpendicular to the straight line passing through the two points : A (– 3 , 4) and B (3 , – 2) 2018 Exam (11) Question (3) (b)
13	If the slope of a straight line equals 2 and the intercepted part from the positive part of y-axis is 6 length unit , **then find :** (1) The equation of this straight line. (2) Its intersection point with x-axis. 2018 Exam (23) Question (3) (b)
14	ABC is a triangle in which $\overline{AB} \perp \overline{BC}$ where A (4 , 1) and B (– 2 , – 1) **Find :** (1) The slope of $\overleftrightarrow{AB}$ (2) The equation of $\overleftrightarrow{BC}$ 2018 Exam (10) Question (5) (b)
15	If $\overline{AB}$ is a diameter in the circle M where B (8 , 11) , M (5 , 7) , **then find :** (1) The coordinates of the point A (2) The length of the radius of the circle. (3) The equation of the perpendicular straight line to $\overline{AB}$ from the point B 2017 Exam (18) Question (5) (a)

16	If the equation of the straight line L_1 is $2x - 3y + a = 0$, and the equation of the straight line L_2 is $3x + by - 6 = 0$, **then find :** (1) The value of b which makes L_1 and L_2 parallel. (2) The value of b which makes L_1 and L_2 perpendicular. (3) The value of a , if the point (1 , 3) lies on L_1 2017 Exam (18) Question (2) (b)
17	Find the slope and the length of the intercepted part from y-axis of the straight line whose equation is : $\frac{x}{3} + \frac{y}{2} = 1$ 2017 Exam (15) Question (5) (b)
18	**In the opposite figure :** $\overleftrightarrow{AB}$ cuts from y-axis a part of length 3 units and AB = 5 length units **Find :** the equation of $\overleftrightarrow{AB}$ y, A, x`, x, O, B, y` 2018 Exam (5) Question (5) (a)
19	**In the opposite figure :** ABO is an equilateral triangle , C is the midpoint of $\overline{AB}$ **Find :** the equation of the straight line $\overleftrightarrow{OC}$ y, A, C, x`, x, O, B, y` 2018 Exam (6) Question (5) (a)
20	2017 Exam (19) Question (4) (a) **In the opposite figure , find :** (1) The slope of $\overleftrightarrow{AB}$ (2) The length of the intercepted part of y-axis. (3) The equation of the straight line $\overleftrightarrow{AB}$ y, B, 2, x`, A, x, O, 3, y`